The Michigan Center for Industrial and Applied Mathematics

Michigan State University

Workshop on Inverse Problems: Theory, Computation, and Applications

Workshop Schedule

Monday, April 19

Session Chair: Gang Bao
0845-0900: Opening Remarks
0900-0945: Michael Vogelius
0945-1030: Yin Zhang

1030-1050: break 1

Session Chair: Satish Udpa
1050-1135: Paul Sacks
1135-1220: Rakesh

1220-1400: lunch (Red Cedar AB)

Session Chair: Andrew Christlieb
1400-1445: John Schotland
1445-1530: Albert Fannjiang

1530-1550: break 2

Session Chair: Andrew Christlieb
1550-1635: Faouzi Triki
1635-1655: KiHyun Yun
1700-1720: Zhengfu Xu

1530-1550: break 3

1740-1900: poster/round table
1900-2100: Dinner (Big Ten C)

Tuesday, April 20

Session Chair: Jianliang Qian
0900-0945: Elena Cherkaev
0945-1030: Chunming Wang

1030-1050: break 4

Session Chair: Patti Lamm
1050-1135: Zuhair Nashed
1135-1220: Eldad Haber

1220-1400: lunch

Session Chair: Jianliang Qian
1400-1445: Kui Ren
1445-1530: Paul Scott Carney
1530-1615: Seraphin Mefire

All sessions will be taken place in Lincoln Room of the Kellogg Center.

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Workshop Description

The Michigan Center for Industrial and Applied Mathematics (MCIAM) at the Michigan State University is pleased to announce a two-day workshop on April 19-20, 2010, on "Inverse Problems: Theory, computation, and applications."

One of the main objectives of the MCIAM center is to expose local, national and international industry to skills and tools that modern applied mathematics can bring to bear on multiscale problems. Problems of interests are non-linear and span many temporal and spatial scales, making these problems challenging from both analytical and numerical perspectives. While modeling has been used as a tool to bridge a wide range of scales, it is the combination of modeling with simulation and analysis that has led to a range of successful tools for exploring these problems; yet there are many problems that remain well beyond the existing capabilities.

This third MCIAM workshop will focus on Theory, Computation, and Applications of Inverse Problems. A main goal of the workshop is to bring together researchers in various fields that involve analysis and computation of inverse problems. The workshop aims to offer a comfortable and spirited gathering at a small scale to promote exchange of ideas between colleagues in computation, analysis, and modeling of inverse problems along with applications, and to explore future research directions.

Organizing Committee:

Gang Bao,	Department of Mathematics, MSU (Workshop Chair)
Andrew Christlieb,	Department of Mathematics, MSU
Patricia Lamm,	Department of Mathematics, MSU
Jianliang Qian,	Department of Mathematics, MSU
Satish Udpa,	College of Engineering, MSU

Invited Speakers:

Paul Scott Carney,	ECE, UIUC
Elena Cherkaev,	Department of Mathematics, University of Utah
Albert Fannjiang,	Department of Mathematics, UC Davis
Eldad Haber,	Mathematics and Computer Science, Emory
Seraphin Mefire,	Universite Henri Poincare, France
Zuhair Nashed,	Department of Mathematics, University of Central Florida
Rakesh,	Department of Mathematical Sciences, University of Delaware
Kui Ren,	Department of Mathematics, University of Texas at Austin
Paul Sacks,	Department of Mathematics, Iowa State University
John Schotland,	Department of Bioengineering, University of Pennsylvania
Faouzi Triki,	Universite Grenoble, France
Michael Vogelius,	Department of Mathematics, University of Rutgers
Chunming Wang,	Department of Mathematics, University of South California
Yin Zhang,	Computational and Applied Mathematics, Rice University
Zhengfu Xu,	Department of Mathematics, Michigan State University
KiHyun Yun,	Department of Mathematics, Michigan State University

Better living through asymptotics: interferometric synthetic aperture microscopy

P. Scott Carney

Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign

Over the last 18 years, optical coherence tomography (OCT) has provided an alternative to physical sectioning and histology that allows for imaging of living samples and even in vivo examination of cell structure and dynamics. Applications range from monitoring the development of engineered tissues to the diagnosis of malignancies. The sectional imaging of OCT is achieved by direct visualization of raw data obtained in focused optical range finding. As a result, there is, in the OCT community, a widely held belief that there exists a trade-off between transverse resolution and the thickness of the volume that may be imaged with a fixed focal plane. The extreme manifestation of this effect may be seen in optical coherence microscopy (OCM) where a single plane is imaged using a highly focused beam to achieve micron scale resolution, but no sectioning is possible because of the defocus away from this plane.

In this talk I will show that solution of the inverse scattering problem leads to algorithms that provide a three-dimensional reconstruction of the object with a spatially invariant pointspread function for the system. We call the new modality interferometric synthetic aperture microscopy (ISAM). The spatial resolution is everywhere equal to the best resolution in the raw data (in the focal plane). Thus the supposed trade-off between resolution and depth of imaging is eliminated. The resultant reconstructions show a marked qualitative improvement in all regions and moreover are quantitatively meaningful.

The algorithms take a particularly simple (and so fast) form by asymptotic evaluation of the forward model in two distinct regimes. I will present the theoretical analysis, numerical simulations and experimental results.

The presented work was done in collaboration with the Boppart group at the University of Illinois.

Inverse problem for the structure of composite materials

Elena Cherkaev

Department of Mathematics, University of Utah

The talk deals with inverse homogenization problem which is a problem of deriving information about the microgeometry of composite material from its effective properties. The approach is based on reconstruction of the spectral measure in the analytic Stieltjes representation of the effective tensor of two-component composite. This representation relates the n-point correlation functions of the microstructure to the moments of the spectral measure, which contains all information about the microgeometry. The problem of identification of the spectral function from effective measurements in an interval of frequency has a unique solution, however the problem is ill-posed. The talk discusses several stabilization techniques as well as Pade approximations used to reconstruct the spectral function. Results of reconstruction of microstructural parameters are shown for visco-elastic composites and for composites of two materials with different complex permittivity. The reconstructed spectral function can be used to characterize the microgeometry and to compute other effective parameters of the same composite; this gives solution to the problem of coupling of different effective properties of a two-component random mixture.

Inverse scattering from the perspective of compressed sensing

Albert Fannjiang

Department of Mathematics, University of California at Davis

Inverse scattering is analyzed from the perspective of compressed sensing. Compressed sensing theory offers potential for reducing the number of measurements for a given target complexity and for efficient reconstruction. In some cases, it is shown that this reduction of measurement complexity exceeds conventional techniques without significant increase in computational complexity.

Design in Inverse Problems

Eldad Haber

Mathematics and Computer Science, Emory University

Structural stability in a minimization problem for conductivity imaging

M. Zuhair Nashed

Department of Mathematics, University of Central Florida

We consider the problem of minimization of the functional $\int_\Omega a(x)|u(x)|dx$ over functions u of bounded variation with prescribed trace f at the boundary. The stability of the minimum value of the functional with respect to the coefficient $a \in L^2(\Omega)$ is established in the vicinity of a coefficient of the form $a = \sigma|u|$, where u solves $\nabla \cdot \sigma \nabla u = 0$ with $u|_{\partial\Omega} = f$. This problem occurs in conductivity imaging when knowledge of the magnitude of the current density field inside a body is available. The method of proof is constructive. This is joint work with Alexandru Tamasan.

Numerical localization of electromagnetic imperfections in 3D bounded domains from a perturbation formula

S.M. Mefire

University Henri Poincare, France

We present numerical localization procedures that result from the combination of a perturbation formula in the electromagnetic fields with suited inversion algorithms. Here, the domains, containing the imperfections in a finite number, are bounded, and the EIT approach is considered in frequency and static contexts.

Some formally determined inverse problems for hyperbolic PDEs – partial results

Rakesh

Department of Mathematical Sciences, University of Delaware

A three dimensional acoustic medium is probed by point sources and the response of the medium is recorded on the boundary of the medium over a certain time interval. From these boundary responses the goal is the recovery of acoustic properties of the medium represented by the coefficient $q(x)$ of the operator $\partial_t^2 - \Delta_x + q(x)$. We describe partial results on uniqueness and stability for three such formally determined inverse problems.

Physics-based models for measurement correlations with applications to inverse problems

Kui Ren

Department of Mathematics, University of Texas at Austin

In many inverse problems, when the forward map is a smoothing (regularizing) operator, the inverse map is usually unbounded. Thus only the low frequency component of the object of interest is accessible from noisy measurements. In many inverse problems however, the neglected high frequency component may significantly affect the measured data. Using simple scaling arguments, we characterize the influence of the high frequency component. We will then show how to eliminate the effect of the high frequency component in a onedimensional inverse spectral problem to obtain better reconstructions of the low frequency component of the unknown. Numerical results with synthetic data will be presented. This is a joint work with Guillaume Bal at Columbia University.

An inverse problems with two wave speeds

Paul Sacks

Department of Mathematics, Iowa State University

We prove uniqueness and stability results for an inverse problem associated with the 2×2 hyperbolic system

$$CU_{tt} = U_{xx} + AU_x + BU$$

on the half space $x > 0$. Here C is a known constant diagonal matrix with distinct positive entries while $A = A(x), B = B(x)$ are to be determined from suitable boundary data. This is joint work with Rakesh, and is closely related to earlier work of Belishev et. al.

Inverse Problems in Quantum Imaging

John C Schotland

University of Pennsylvania

I will review recent work on inverse scattering problems that arise in imaging with entangled photons. Experiments involving entanglement via post-detection selection or due to illumination with down converted photons will be analyzed.

Inverse Medium Problem with Internal Data

Faouzi Triki

Laboratoire Jean Kuntzmann, Universite Joseph Fourier, B.P. 53, 38041 Grenoble Cedex 9

Let $\Omega \subset \mathbb{R}^d$, $d = 2, 3$ be a bounded open and connected domain with boundary of class C^2 and $q(x)$ be a real function that lies in $L^\infty(\Omega)$. For a boundary data $g \in H^{\frac{1}{2}}(\Omega)$, we consider the following Helmholtz equation

$$\begin{aligned} \Delta u + qu &= 0 && \Omega, \\ u &= g && \partial\Omega. \end{aligned} \tag{1}$$

We assume that q satisfies the following assumptions

- there exists a constant $\mu > 0$ such that $\mu \le q(x)$, for a.e. $x \in \Omega$.

- the homogeneous problem with $g = 0$ admits the unique solution $u = 0$, so that 0 is not in the spectrum of $\Delta + q$.

Under these assumptions it is known that there exists a unique $u \in H^1(\Omega)$, solution to the problem (**??**). The internal data are in the form

$$I_g(q) = qu^2. \tag{2}$$

The inverse medium problem with internal data consists of reconstructing $q(x)$ in Ω from knowledge of $I_g(q)$ for a given Dirichlet data g. The data $I_g(q)$ represents a density of energy and can be retrieved by small elastic perturbations. These techniques are also used for electrical impedance tomography.

In this paper we first show that knowledge of one real internal data uniquely determines the medium. A local Lipschitz stability of the reconstruction is also derived. This result is in contrast with inverse medium problem with boundary data, where the ill-posdness is severe and the stability is logarithmic.

Approximate cloaking at all frequencies

Michael Vogelius

Department of Mathematics, Rutgers, The State University of New Jersey

I shall describe some recent joint work with Kohn, Onofrei and Weinstein and with Nguyen, the focus of which has been the (design and the) assessment of the efficiency of a very natural approximate cloaking by (mapping) scheme for the Helmholtz equation.

Estimation of Driving Forces in Ionospheric Data Assimilation Model

Chunming Wang

Department of Mathematics, University of South California

As a part of the Earth's atmosphere the ionosphere becomes increasingly more important to human activities due to the wide use of wireless communication. The monitoring and forecast of the Earth's ionospheric condition require accurate determination of key parameters, also known as, driving forces in the mathematical model of the ionospheric plasma dynamics. In a collaborative

effort between the Jet Propulsion Laboratory (JPL) and the University of Southern California, we have developed a Global Assimilative Ionospheric Model (GAIM) that is capable to estimating these ionospheric driving forces. In this talk, we shall present the mathematical approach known as 4DVAR for the estimation of these forces. We shall also present the implementation and validation efforts we have made in the development of GAIM.

Stability for an inverse problem related to a plasma wave equation

KiHyun Yun

Department of Mathematics, Michigan State University

We consider an inverse problem of determining an unknown potential q in a plasma wave equation via the Neumann to Dirichlet map. Until now, the uniqueness theorem and a Holder type stablity result with exponent $\frac{1}{3} - \epsilon$ for the inverse problem have been establised, based on the technique inspired by Sylverster and Uhlmann. In this talk, an effective way to utilize the technique is presented to obtain an improved nearly Lipschitz type stability estimate under the same condition.

Sparsity Regularizations and Augmented Lagrangian Alternating Direction Methods

Yin Zhang

Computational and Applied Mathematics, Rice University

We will introduce our recent works on applying augmented Lagrangian based alternating direction methods to solving problems in imaging reconstruction and compressive sensing where sparsity is the crucial prior information.