

11

$$6 \text{ (e)} \begin{cases} 2x_1 + x_2 + 3x_3 = 1 \\ 4x_1 + 3x_2 + 5x_3 = 1 \\ 6x_1 + 5x_2 + 5x_3 = -3 \end{cases}$$

Sol: The augmented matrix

$$\left( \begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 3 & 5 & 1 \\ 6 & 5 & 5 & -3 \end{array} \right)$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left( \begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -4 & -6 \end{array} \right)$$

$$\begin{array}{l} R_1 - R_2 \\ R_3 - 2R_2 \end{array} \left( \begin{array}{ccc|c} 2 & 0 & 4 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -4 \end{array} \right)$$

$$\begin{array}{l} R_1 + 2R_3 \\ R_2 - \frac{1}{2}R_3 \end{array} \left( \begin{array}{ccc|c} 2 & 0 & 0 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & -4 \end{array} \right)$$

$$\begin{array}{l} R_1/2 \\ R_3/-2 \end{array} \left( \begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

R. R. E. form

$$\begin{cases} x = -3 \\ y = 1 \\ z = 2 \end{cases}$$

$(-3, 1, 2)$

1.2

$$3(d) \left( \begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 & 4 \end{array} \right)$$

sol:

$$\left( \begin{array}{cc|cc|c} x & y & z & w & \\ \hline 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 & 4 \end{array} \right)$$

↑ free      ↑ free

$$\text{Let } y = \alpha \quad w = \beta$$

$$\text{then } x = 5 - 2\alpha - \beta$$

$$z = 4 - 3\beta$$

$$\text{Sol. set} = \{ (5 - 2\alpha - \beta, \alpha, 4 - 3\beta, \beta) \mid \alpha, \beta \in \mathbb{R} \} \quad \#$$

$$8. \text{ sol: } \left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array} \right) \xrightarrow[R_3 - 2R_1]{R_2 + R_1} \left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & -6 & a-2 & 1 \end{array} \right) \xrightarrow{R_3 + R_2} \left( \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & 0 & a+2 & 4 \end{array} \right)$$

The system has a unique sol  
if and only if  $a+2 \neq 0$  ( $a \neq -2$ ). #

1.3

$$1(g) \quad A^T = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 0 & 2 \\ 4 & 1 & 2 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 2 & 1 & 1 \end{pmatrix}$$

$$A^T B^T = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 0 & 2 \\ 4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 2 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3+0+2 & -9-2+1 & 6+8+1 \\ 1+0+4 & -3+0+2 & 2+0+2 \\ 4+0+4 & -12+1+2 & 8-4+2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{pmatrix} \#$$

$$4(c) \quad \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \#$$