§ 4.2

5 (a) $A = R_{-\frac{\pi}{4}} = \begin{pmatrix} \cos(-\frac{\pi}{4}) & -\sin(-\frac{\pi}{4}) \\ \sin(-\frac{\pi}{4}) & \cos(-\frac{\pi}{4}) \end{pmatrix}$

$= \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

(b) $A = R_{\frac{\pi}{2}} R_x = \begin{pmatrix} \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(c) $A = R_{\frac{\pi}{6}} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \cos(\frac{\pi}{6}) & -\sin(\frac{\pi}{6}) \\ \sin(\frac{\pi}{6}) & \cos(\frac{\pi}{6}) \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}$

(d) $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} R_D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

14 Sol. Let $C = \{x^2, x, 1\}$ $D = \{2, 1-x\}$

$L(x^2) = (x^2)' + 0 = 2x = 1\cdot 2 + (-2)(1-x)$

$\Rightarrow [L(x^2)]_D = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$L(x) = x' + 0 = 1 = \frac{1}{2}\cdot 2 + 0\cdot(1-x)$

$\Rightarrow [L(x)]_D = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$

$L(1) = 1' + 1 = 1 = \frac{1}{2}\cdot 2 + 0(1-x)$

$\Rightarrow [L(1)]_D = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$

Under the bases C & D the linear transformation L is given by $A = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -2 & 0 & 0 \end{pmatrix}$

(a) $[L(x^2+2x-3)]_D = A\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -2 \end{pmatrix}$

(b) $[L(x^2+1)]_D = A\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ -2 \end{pmatrix}$

(c) $[L(3x)]_D = A\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}$

(d) $[L(4x^2+2x)]_D = A\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$

4.3

5 Sol: (a) Let $C = \{1, x, x^2\}$

$L(1) = 0 \Rightarrow [L(1)]_C = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$L(x) = x \cdot x' + x'' = x \Rightarrow [L(x)]_C = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$L(x^2) = x \cdot (x^2)' + (x^2)'' = 2x^2 + 2 \Rightarrow [L(x^2)]_C = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

Thus $A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(b) Let $D = \{1, x, 1+x^2\}$

$[L(1)]_D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad [L(x)]_D = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$L(1+x^2) = x \cdot (1+x^2)' + (1+x^2)'' = 2x^2+2 = 2(x^2+1)$

$\Rightarrow [L(1+x^2)]_D = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

Thus $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(c) Note $B = U_{CD}^{-1} A U_{CD}$

where $U_{CD} = \left([1]_C \ [x]_C \ [1+x^2]_C \right) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Thus we can take $S = U_{CD} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\left[L^n(P(x))\right]_D = B^n [P(x)]_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ a_1 \\ 2^n a_2 \end{pmatrix}$

Thus $L^n(P(x)) = 0 \cdot 1 + a_1 \cdot x + 2^n a_2 \cdot (1+x^2)$

$= 2^n a_2 x^2 + a_1 x + 2^n a_2$

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