§ 5.1

1. (a) $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|}$

$= \dfrac{(2,1,3)^T \cdot (6,3,9)^T}{\sqrt{2^2+1^2+3^2}\sqrt{6^2+3^2+9^2}}$

$= \dfrac{12+3+27}{42}$

$= 1$

$\theta = 0$ #

3. (c) $\vec{p} = \mathrm{Proj}_{\vec{y}}\vec{x} = \dfrac{\vec{x}\cdot\vec{y}}{\|\vec{y}\|^2}\vec{y}$

$= \dfrac{(2,4,3)^T \cdot (1,1,1)^T}{1^2+1^2+1^2}(1,1,1)^T$

$= (3,3,3)^T$

$\vec{x} - \vec{p} = (2,4,3)^T - (3,3,3)^T = (-1,1,0)^T$

$\vec{p}\cdot(\vec{x}-\vec{p}) = (3,3,3)^T \cdot (-1,1,0)^T = -3+3+0 = 0$

Thus $\vec{p} \perp \vec{x}-\vec{p}$ #

7. Sol: Let $\vec{v} = (1, 2)^T$ $\vec{w} = (4, -3)^T$

denote the line $4x - 3y = 0$ as L.

For any $\vec{z} = (x, y) \in L$, we have

$\vec{z} \cdot \vec{w} = 0$ that is $\vec{z} \perp \vec{w}$

Therefore

$$\text{dist}(\vec{v}, L) = \left| \text{comp}_{\vec{w}} \vec{v} \right|$$

$$= \left| \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|} \right|$$

$$= \left| \frac{(1, 2)^T \cdot (4, -3)^T}{\sqrt{4^2 + (-3)^2}} \right|$$

$$= \frac{2}{5}$$

#

18 (a) Proof: $\vec{p} \cdot \vec{z} = \left(\frac{\vec{x}^T \vec{y}}{\vec{y}^T \vec{y}} \vec{y} \right) \cdot \left(\vec{x} - \frac{\vec{x}^T \vec{y}}{\vec{y}^T \vec{y}} \vec{y} \right)$

$$= \frac{(\vec{x}^T \vec{y})^2}{\vec{y}^T \vec{y}} - \left(\frac{\vec{x}^T \vec{y}}{\vec{y}^T \vec{y}} \right)^2 \vec{y}^T \vec{y}$$

$$= 0$$

Thus, $\vec{p} \perp \vec{z}$

(b) Sol: $\|\vec{x}\|^2 = \|\vec{p} + \vec{z}\|^2 = (\vec{p} + \vec{z}) \cdot (\vec{p} + \vec{z})$

$$= \|\vec{p}\|^2 + \|\vec{z}\|^2 + 2 \vec{p} \cdot \vec{z}$$

$$= 6^2 + 8^2 + 2 \cdot 0 = 100$$

Thus, $\|\vec{x}\| = 10$.

20. Sol:

$$\vec{x}_1 \overset{\circ}{=} \begin{pmatrix} 61 \\ 63 \\ 78 \\ 65 \\ 63 \end{pmatrix} \quad \vec{d}_1 = \vec{x}_1 - \overline{\vec{x}_1} = \begin{pmatrix} -5 \\ -3 \\ 12 \\ -1 \\ -3 \end{pmatrix}$$

$$\vec{x}_2 \overset{\circ}{=} \begin{pmatrix} 53 \\ 73 \\ 61 \\ 84 \\ 59 \end{pmatrix} \quad \vec{d}_2 = \vec{x}_2 - \overline{\vec{x}_2} = \begin{pmatrix} -13 \\ 7 \\ -5 \\ 18 \\ -7 \end{pmatrix}$$

$$\vec{x}_3 \overset{\circ}{=} \begin{pmatrix} 53 \\ 78 \\ 82 \\ 96 \\ 71 \end{pmatrix} \quad \vec{d}_3 = \vec{x}_3 - \overline{\vec{x}_3} = \begin{pmatrix} -23 \\ 2 \\ 6 \\ 20 \\ -5 \end{pmatrix}$$

The correlation matrix

$$D = \begin{pmatrix} \dfrac{\vec{d}_1 \cdot \vec{d}_1}{\|\vec{d}_1\|^2} & \dfrac{\vec{d}_1 \cdot \vec{d}_2}{\|\vec{d}_1\|\|\vec{d}_2\|} & \dfrac{\vec{d}_1 \cdot \vec{d}_3}{\|\vec{d}_1\|\|\vec{d}_3\|} \\ \dfrac{\vec{d}_1 \cdot \vec{d}_2}{\|\vec{d}_1\|\|\vec{d}_2\|} & \dfrac{\vec{d}_2 \cdot \vec{d}_2}{\|\vec{d}_2\|^2} & \dfrac{\vec{d}_2 \cdot \vec{d}_3}{\|\vec{d}_2\|\|\vec{d}_3\|} \\ \dfrac{\vec{d}_1 \cdot \vec{d}_3}{\|\vec{d}_1\|\|\vec{d}_3\|} & \dfrac{\vec{d}_2 \cdot \vec{d}_3}{\|\vec{d}_2\|\|\vec{d}_3\|} & \dfrac{\vec{d}_3 \cdot \vec{d}_3}{\|\vec{d}_3\|^2} \end{pmatrix}$$

$$\approx \begin{pmatrix} 1.00 & -0.04 & 0.41 \\ -0.04 & 1.00 & 0.87 \\ 0.41 & 0.87 & 1.00 \end{pmatrix}$$

which shows that (i) English & Math are almost NOT correlated,

(ii) Math & Science are positively correlated,

so are Eng & Sci with a weaker correlation.