Math 421 / Homework 11.2

 # 3 Prove that $f(x,y) = \sqrt{|xy|}$ is not differentiable at $(0,0)$.

 # 5 Prove that

$$f(x,y) = \begin{cases} \dfrac{x^4+y^4}{(x^2+y^2)^\alpha} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is differentiable on $\mathbf{R}^2$ for all $\alpha < 3/2$.

 # 7 Prove that

$$f(x,y) = \begin{cases} \dfrac{x^3-xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is continuous on $\mathbf{R}^2$ and has first-order partial derivatives everywhere on $\mathbf{R}^2$, but f is not differentiable at $(0,0)$.

 # 9 Let $r > 0$, $f\colon B_r(\mathbf{0}) \to \mathbf{R}$, where $B_r(\mathbf{0})$ is an open ball centered at $\mathbf{0}$ in $\mathbf{R}^n$, and suppose that there exists an $\alpha > 1$ such that $|f(\mathbf{x})| \leq \|\mathbf{x}\|^\alpha$ for all $\mathbf{x} \in B_r(\mathbf{0})$. Prove that f is differentiable at $\mathbf{0}$. What happens to this result when $\alpha = 1$?