Math 421 / Homework 5.2

 # 1 Use the connection between integrals and areas, evaluate each of the following integrals.

(a)

$$\int_{-2}^{2} |x+1| dx$$

(b)

$$\int_{-2}^{2} (|x+1| + |x|) dx$$

(c)

$$\int_{-a}^{a} \sqrt{a^2 - x^2}\, dx$$

 # 7 Suppose that f is integrable on $[a, b]$, that $x_0 = a$, and that (x_n) is a sequence of numbers in $[a, b]$ such that $x_n \uparrow b$ as $n \to \infty$. Prove that

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{k=0}^{n} \int_{x_k}^{x_{k+1}} f(x)\, dx.$$

 # 9 Let $f\colon [a, b] \to \mathbf{R}$, $a = x_0 < x_1 < \cdots < x_n = b$, and suppose that $f(x_k+)$ exists and is finite for $k = 0, 1, \cdots, n-1$ and $f(x_k-)$ exists and is finite for $k = 1, 2, \cdots, n$. Show that if f is continuous on each subinterval (x_{k-1}, x_k), then f is integrable on $[a, b]$ and

$$\int_a^b f(x)\, dx = \sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} f(x)\, dx.$$