

Name:

# ANSWER KEY

Math 132: Section 21

Midterm 3

November 16, 2009

The exam is all partial credit. Please write neatly and clearly, showing all of your work. No calculators, cell phones, books, or notes may be used. The test contains 100 possible points. Good luck!

1	
2	
3	
4	
5	
6	
7	
Total	

1. (15 points) Find the absolute maximum and minimum of  $f(x) = x^3 - 3x^2 + 1$  on the interval  $[-2, 1]$ .

$$f'(x) = 3x^2 - 6x$$

$$0 = f'(x) = 3x(x-2)$$

$$x = 0, \underline{2} \\ \text{not in } [-2, 1]$$

Plug in crit pts and endpoints

$$f(-2) = (-2)^3 - 3(-2)^2 + 1 = -19 \quad \leftarrow \min$$

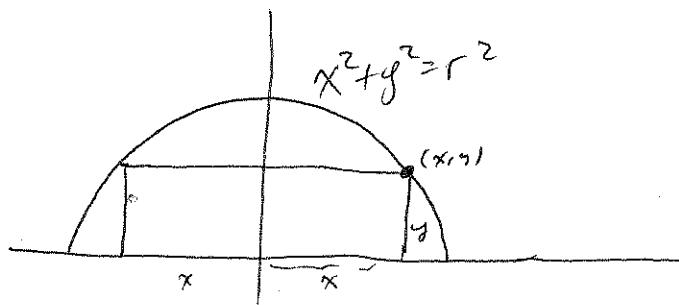
$$f(0) = 1$$

$$f(1) = -1 \quad \leftarrow \max$$

Absolute Min at  $(-2, -19)$

Abs Max at  $(0, 1)$

2. (20 points) Find the area of the largest rectangle that can be inscribed in a semicircle of radius  $r$ .



$$A = 2xy$$

$$x^2 + y^2 = r^2$$

$$y = \sqrt{r^2 - x^2}$$

$$\underline{A(x) = 2x\sqrt{r^2 - x^2}}$$

← Maximize this

$$A' = 2\sqrt{r^2 - x^2} + 2x \frac{1}{2} (r^2 - x^2)^{-\frac{1}{2}} (-2x)$$

$$0 = 2\sqrt{r^2 - x^2} - 2x^2 \frac{1}{\sqrt{r^2 - x^2}}$$

$$\frac{x^2}{\sqrt{r^2 - x^2}} = \sqrt{r^2 - x^2}$$

$$x^2 = r^2 - x^2$$

$$x^2 = \frac{r^2}{2}$$

$$\underline{x = \frac{r}{\sqrt{2}}}$$

$$\Leftrightarrow \Rightarrow y = \sqrt{r^2 - \left(\frac{r}{\sqrt{2}}\right)^2} \\ = \sqrt{r^2 - \frac{r^2}{2}} = \frac{r}{\sqrt{2}}$$

$$\Rightarrow \boxed{A\left(\frac{r}{\sqrt{2}}\right) = 2\left(\frac{r}{\sqrt{2}}\right)\left(\frac{r}{\sqrt{2}}\right) = r^2}$$

3. (5 points) Using the fact that  $\frac{d}{dx}(\sin(x)) = \cos(x)$ , show that there exists some  $c$  in the closed interval  $[0, \frac{\pi}{2}]$  such that  $\cos(c) = \frac{2}{\pi}$ .

Let  $f(x) = \sin x$ , which is diff'l.

Mean Value Theorem implies exists  $c \in [0, \frac{\pi}{2}]$  st

$$\frac{f(\frac{\pi}{2}) - f(0)}{\frac{\pi}{2} - 0} = f'(c)$$

" "

$$\frac{\sin(\frac{\pi}{2}) - \sin(0)}{\frac{\pi}{2}} = \cos(c)$$

$$\frac{1}{\frac{\pi}{2}} = \frac{2}{\pi} = \cos(c)$$

" " "

4. (15 points) Consider the function  $f(x) = \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2$ .

(a) Find where  $f(x)$  is increasing and decreasing and the  $x$ -values of any local extrema.

(b) Find where  $f(x)$  is concave up and concave down and the  $x$ -values of any inflection points.

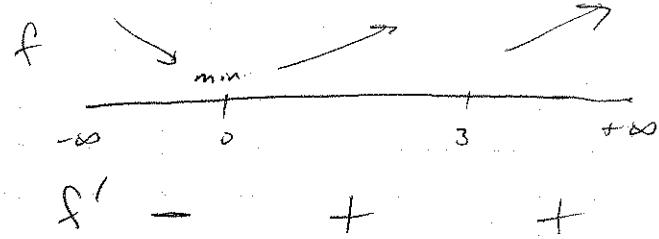
(c) Sketch a graph of  $f(x)$ . Make sure your graph reflects the information obtained above.

a.  $f' = x^3 - 6x^2 + 9x$

$$0 = f' = x(x^2 - 6x + 9)$$

$$0 = x(x-3)^2$$

$$\underline{x = 0, 3}$$



Inc  $(0, 3), (3, +\infty)$

Dec  $(-\infty, 0)$

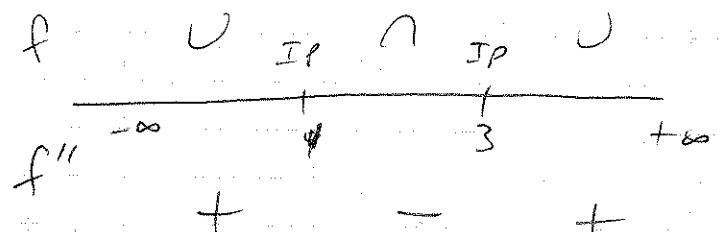
Local min  $x=0$

b.  $f'' = 3x^2 - 12x + 9$

$$0 = 3(x^2 - 4x + 3)$$

$$0 = 3(x-1)(x-3)$$

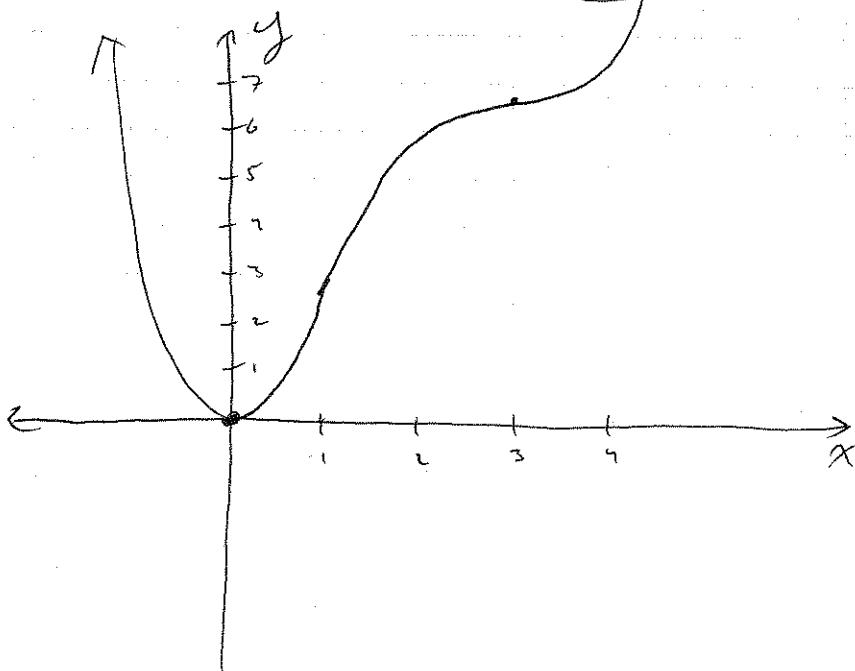
$$\underline{x = 1, 3}$$



f Conc Up  $(-\infty, 1), (3, +\infty)$

Conc Down  $(1, 3)$

IP at  $x = 1, 3$



5. (15 points) Compute the following limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x - 1} = (\text{Plug-in} = \frac{1^3 - 3(1) + 2}{1 - 1} = \frac{0}{0})$$

$$\begin{aligned} \text{L'Hopital's} \\ \lim_{x \rightarrow 1} \frac{3x^2 - 3}{1} &= \frac{3(1)^2 - 3}{1} = \boxed{0} \end{aligned}$$

$$(b) \lim_{x \rightarrow 0} \frac{\cos(2x) - \cos(x)}{x^2} = (\text{Plug-in} = \frac{\cos 0 - \cos 0}{0^2} = \frac{0}{0})$$

$$\begin{aligned} \text{L'Hopital} \\ \lim_{x \rightarrow 0} \frac{-2\sin(2x) + \sin(x)}{2x} &\quad (\text{Plug-in} = \frac{-2\sin 0 + \sin 0}{0} = \frac{0}{0}) \end{aligned}$$

$$\begin{aligned} \text{L'Hopital} \\ \lim_{x \rightarrow 0} \frac{-4\cos(2x) + \cos(x)}{2} &= \frac{-4\cos 0 + \cos 0}{2} \\ &= \frac{-4+1}{2} = \boxed{-\frac{3}{2}} \end{aligned}$$

6. (15 points) (a). You wish to find a solution to the equation  $x^2 + x - 1 = 0$ . Use Newton's method with  $x_0 = 0$  and find  $x_1, x_2$ .

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f = x^2 + x - 1$$

$$f' = 2x + 1$$

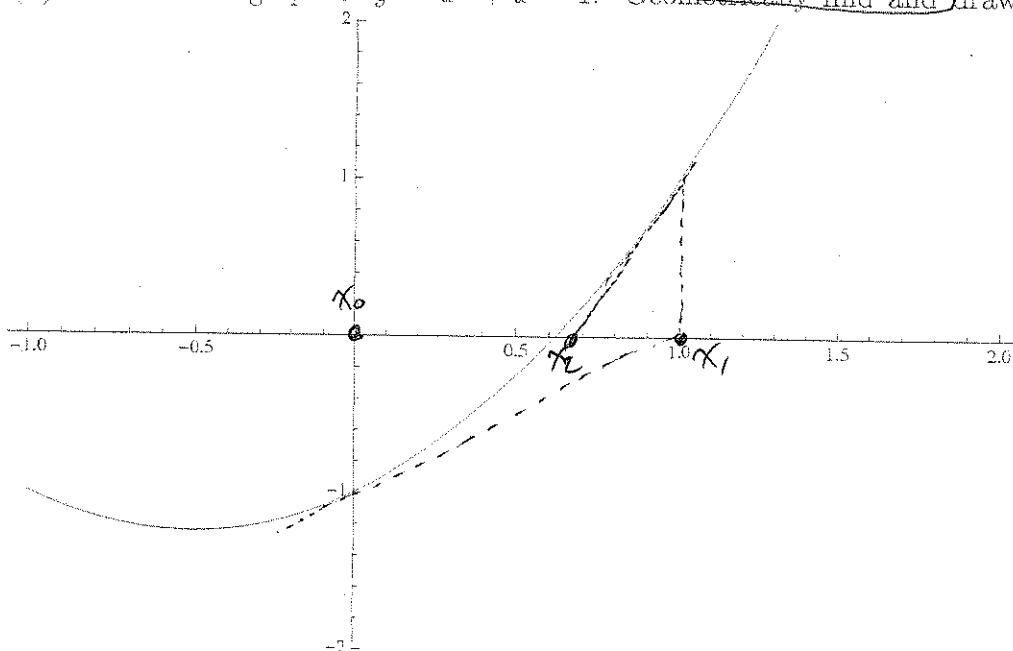
$$x_0 = 0$$

$$x_1 = 0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{-1}{1} = 1$$

$$x_2 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1}{3} = 2/3$$

$$\boxed{x_0 = 0, \quad x_1 = 1, \quad x_2 = 2/3}$$

(b). Below is a graph of  $y = x^2 + x - 1$ . Geometrically find and draw  $x_0, x_1, x_2$  on the graph.



7. (15 points) Find the following anti-derivatives:

$$(a) \int (x^4 - 2x^2 + 3)dx = \frac{1}{5}x^5 - \frac{2}{3}x^3 + 3x + C$$

$$(b) \int \frac{1}{\sqrt[3]{x}} dx = \int x^{-1/3} dx = \frac{3}{2}x^{2/3} + C$$

$$(c) \int (\cos(2x) + \sin(2x)) dx = \frac{1}{2}\sin(2x) - \frac{1}{2}\cos(2x) + C$$