

Name: Answer Key

# Math 132: Section 21

## Midterm 4

December 9, 2009

The exam is all partial credit. Please write neatly and clearly, showing all of your work. No calculators, cell phones, books, or notes may be used. The test contains 100 possible points. Good luck!

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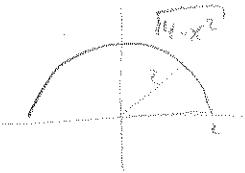
1. (30 points) Compute the following integrals and/or derivatives:

a.  $\int (x^5 - 3x^2 + 2) dx = \boxed{\frac{1}{6}x^6 - x^3 + 2x + C}$

b.  $\int_0^3 2x^2 dx = \frac{2}{3}x^3 \Big|_0^3 = \frac{2}{3}(3)^3 = 2 \cdot 3^2 = \boxed{18}$

c.  $\int \sec^2(3x) dx = \boxed{\frac{1}{3} \tan(3x) + C}$

d.  $\int_0^2 5\sqrt{4-x^2} dx = 5 \int_0^2 \sqrt{4-x^2} dx = 5 \left[ \frac{1}{4} \text{Area circle rad } 2 \right]$   
 $= 5 \left( \frac{1}{4} \pi (2)^2 \right) = \boxed{5\pi}$



e.  $\int 20 \sin^4(2x) \cos(2x) dx$

Let  $u = \sin(2x)$   
 $du = 2 \cos(2x) dx$   
 $\frac{1}{2} du = \cos(2x) dx$

$$\int 20u^4 \frac{1}{2} du = \int 10u^4 du = 2u^5 + C = \boxed{2 \sin^5(2x) + C}$$

2. (30 points) Compute the following integrals and/or derivatives:

a.  $\int_0^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$

let  $u = \sqrt{x}$   
 $du = \frac{1}{2\sqrt{x}} dx$   
 $2du = \frac{1}{\sqrt{x}} dx$

$x=0 \Rightarrow u=0$   
 $x=\pi^2 \Rightarrow u=\sqrt{\pi^2}=\pi$

$$\begin{aligned} \int_0^{\pi} 2 \sin u du &= -2 \cos u \Big|_0^{\pi} = -2[\cos \pi - \cos 0] \\ &= -2[-1 - 1] = (-2)(-2) = \boxed{4} \end{aligned}$$

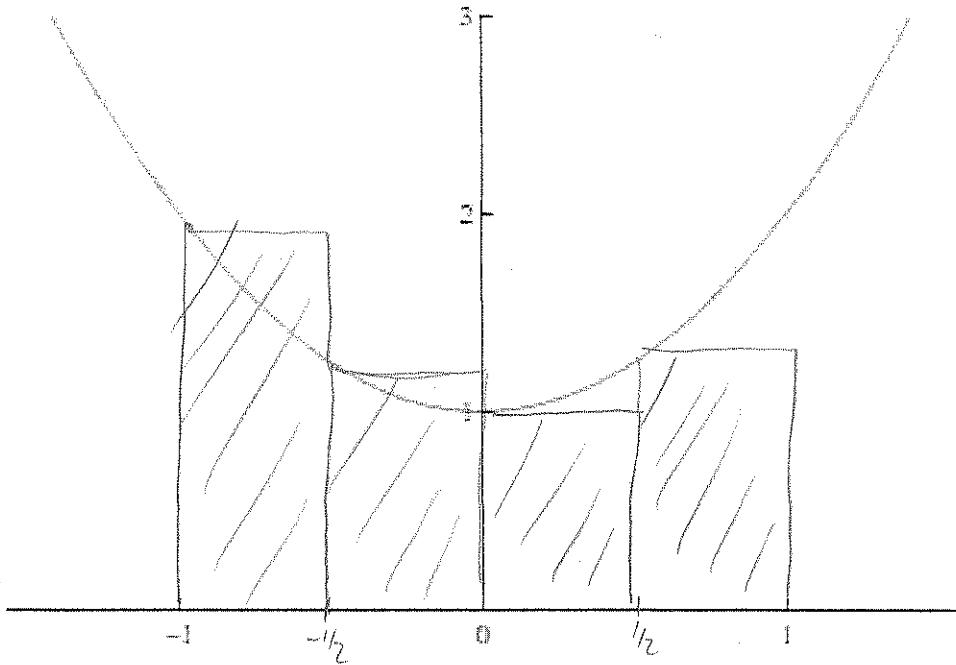
b.  $\int_0^{\pi/2} (f(\sin x) \cos x + x) dx$ , given that  $\int_0^1 f(x) dx = \frac{1}{2}$

$$\begin{aligned} &= \int_0^{\pi/2} f(\sin x) \cos x dx + \int_0^{\pi/2} x dx \\ &\quad \text{let } u = \sin x \\ &\quad du = \cos x dx \\ &\quad x=0 \Rightarrow u = \sin 0 = 0 \\ &\quad x=\pi/2 \Rightarrow u = \sin \pi/2 = 1 \\ &= \int_0^1 f(u) du + \left[ \frac{1}{2} x^2 \right]_0^{\pi/2} = \frac{1}{2} + \frac{1}{2} (\pi/2)^2 = \boxed{\frac{1}{2} + \frac{\pi^2}{8}} \\ &c. \frac{d}{dx} \int_{\pi}^x \tan(t) dt = \boxed{\tan x} \end{aligned}$$

d.  $\frac{d}{dx} \int_{-2x}^{x^3} \frac{d\theta}{\sqrt{1-2\sin^2 \theta}}$

$$= \left[ \frac{3x^2}{\sqrt{1-2\sin^2(x^3)}} - \frac{-2}{\sqrt{1-2\sin^2(-2x)}} \right]$$

3. (10 points) Estimate the integral  $\int_{-1}^1 (x^2 + 1) dx$  using 4 rectangles and left-endpoints. Draw your estimation on the graph below.



$$\text{Let } f = x^2 + 1, \quad \Delta x = \frac{1-(-1)}{4} = \frac{1}{2}$$

$$\int_{-1}^1 (x^2 + 1) dx \approx [f(-1) + f(-\frac{1}{2}) + f(0) + f(\frac{1}{2})] \Delta x$$

$$= [(-1)^2 + 1 + (-\frac{1}{2})^2 + 1 + 0^2 + 1 + (\frac{1}{2})^2 + 1] \frac{1}{2}$$

$$= [4 + 1 + \frac{1}{2}] \frac{1}{2} = \boxed{\frac{11}{4}}$$

4. (10 points) Use the definition of the definite integral to compute  $\int_0^3 2x^2 dx$ .

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x, \quad \text{where } \Delta x = \frac{b-a}{n}, \quad x_k = a + k \Delta x$$

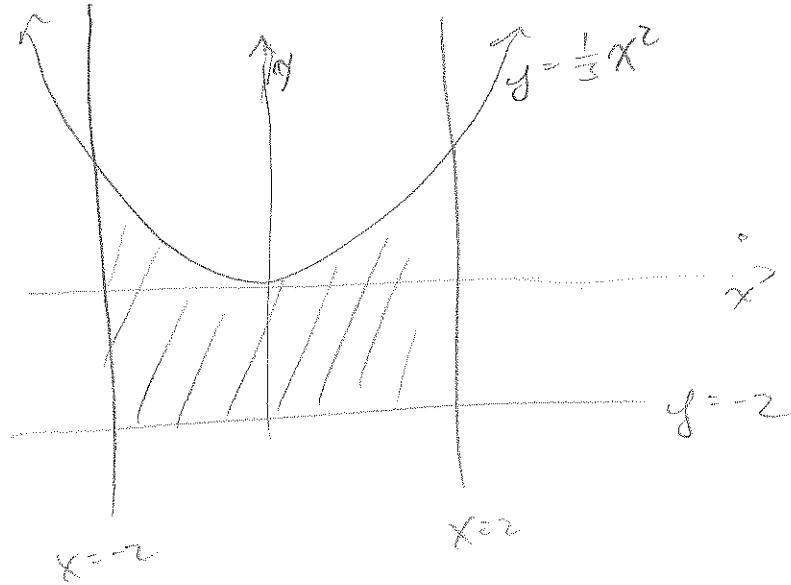
$$\text{Here, } \Delta x = \frac{3}{n}, \quad x_k = 0 + k \frac{3}{n} = \frac{3k}{n}$$

$$\begin{aligned} \sum_{k=1}^n f(x_k) \Delta x &= \sum_{k=1}^n 2\left(\frac{3k}{n}\right)^2 \frac{3}{n} = \sum_{k=1}^n \frac{2 \cdot 3^3}{n^3} k^2 \\ &= \frac{2 \cdot 3^3}{n^3} \sum_{k=1}^n k^2 = \frac{2 \cdot 3^3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \end{aligned}$$

$$\begin{aligned} \int_0^3 2x^2 dx &= \lim_{n \rightarrow \infty} \sum_{k=1}^n 2\left(\frac{3k}{n}\right)^2 \frac{3}{n} = \lim_{n \rightarrow \infty} \frac{2 \cdot 3^3}{6} \cdot \frac{n(n+1)(2n+1)}{n^3} \\ &= \frac{2 \cdot 3^3}{6} \cdot 2 \\ &= 2 \cdot 3^2 = \boxed{18} \end{aligned}$$

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5. (10 points) Find the area between the curves  $y = -2$ ,  $y = \frac{1}{3}x^2$ ,  $x = -2$ ,  $x = 2$ .



$$\begin{aligned}
 A &= \int_{-2}^2 \left( \frac{1}{3}x^2 - (-2) \right) dx = \int_{-2}^2 \frac{1}{3}x^2 + 2 \, dx \\
 &= 2 \left[ \left( \frac{1}{3}x^3 + 2x \right) \Big|_0^2 \right] \\
 &= 2 \left( \frac{2^3}{3} + 2 \cdot 2 \right) = \frac{16}{9} + 8 = \frac{88}{9}
 \end{aligned}$$

6. (10 points) Find the area bounded between the curves  $y = x^3 + 6x$  and  $y = 5x^2$ . You may leave your answer in the form of a definite integral.

Where do curves intersect?

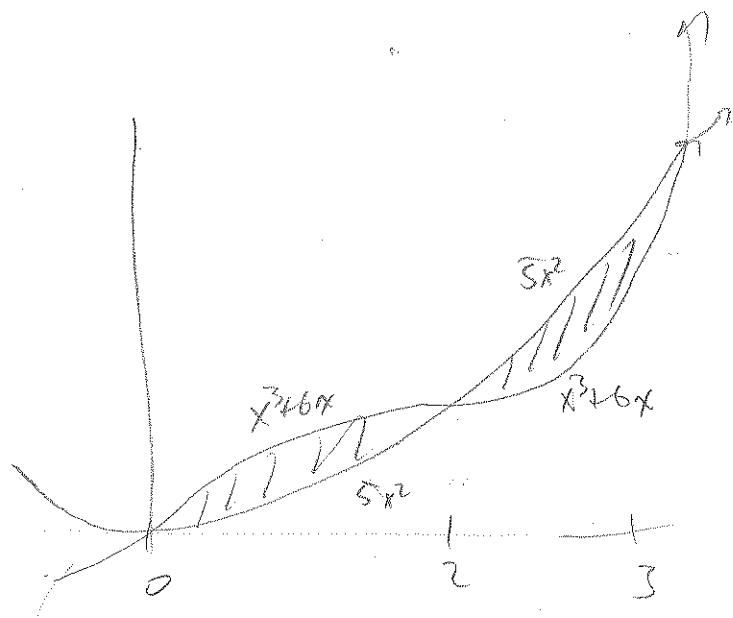
$$x^3 + 6x = 5x^2$$

$$x^3 - 5x^2 + 6x = 0$$

$$x(x^2 - 5x + 6) = 0$$

$$x(x-2)(x-3) = 0$$

$$x = 0, 2, 3$$



Check: Which curve is on top/bottom  
by plugging in sample values.

$$(1)^3 + 6(1) - 5(1)^2 = 7 > 0 \Rightarrow x^3 + 6x \text{ or top on } [0, 2]$$

or  
 $\circ (1)(1-2)(1-3) = + - - \Rightarrow$

Since plug in 2.5

$$(2.5)^3 + 6(2.5) - 5(2.5)^2 = + + - = -$$

$$\Rightarrow 5x^2 \text{ or top on } [2, 3]$$

$$\text{Area} = \int_0^2 x^3 + 6x - 5x^2 \ dx + \int_2^3 5x^2 - (x^3 + 6x) \ dx$$