HW ASSIGNED FRIDAY SEPTEMBER 23

(1) (1.8: 11) Determine whether $S = \{f \in \mathbb{D}(\mathbb{R}) | f'(x) + f(x) = 1 \text{ for all } x \in \mathbb{R}\}$ is a subspace of $\mathbb{D}(\mathbb{R})$.

(2) (1.8: 12) Determine whether $S = \{f \in \mathbb{D}(\mathbb{R}) \mid 2f'(x) + x^2 f(x) = 0 \text{ for all } x \in \mathbb{R}\}$ is a subspace of $\mathbb{D}(\mathbb{R})$.

(3) Let S be the solution set of a second order homogenous linear differential equation

$$a(x)f'' + b(x)f' + c(x)f = 0,$$

where $f = f(x) \in \mathbb{D}^{(2)}(\mathbb{R})$ is the unknown, and $a, b, c \in \mathbb{F}(\mathbb{R})$; i.e. S is all functions f satisfying the equation above. Show that S is a subspace of $\mathbb{D}^{(2)}(\mathbb{R})$.

(4) (1.8: 20) Find subspaces S and T of $\mathbb{R}^2$ such that the union $S \cup T$ is not a subspace of $\mathbb{R}^2$.

Definition 1. Let W be a vector space and $V \subset W$ a subspace. A subset $A \subset W$ is an *affine subspace* over V if:

(a) A is closed under addition from V; i.e. if $\mathbf{w} \in A, \mathbf{v} \in V$, then $\mathbf{w} + \mathbf{v} \in A$.

(b) If $\mathbf{w}_1, \mathbf{w}_2 \in A$, then $\mathbf{w}_2 - \mathbf{w}_1 \in V$.

(5) Show that the set $S = \{(x, y) \in \mathbb{R}^2 \mid 2x + y = 1\} \subset \mathbb{R}^2$ is an affine subspace over $V = \{(x, y) \in \mathbb{R}^2 \mid 2x + y = 0\} \subset \mathbb{R}^2$.