Test 1 Practice Test

Math 309, Section 3

1. Consider the following system of linear equations:

$$\begin{aligned} -3y + z &= 1 \\ x + y - 2z &= 2 \\ x - 2y - z &= 3 \end{aligned}$$

   Write the coefficient matrix associated to the linear system. Use Gaussian elimination (and write what elementary row operations you use) to put the matrix into reduced echelon form. Write the solution set to the system of linear equations.

2. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a finite subset of the vector space V. Write the definition of $\operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. What does it mean for $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to span V? Give the definition of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ being linearly independent. By definition, what does it mean for $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to be a basis?

3. Is $\{\mathbf{0}\}$ linearly independent? Justify your answer.

4. (a) Suppose that $\{\mathbf{v}, \mathbf{w}\}$ are linearly independent in V, and $\mathbf{x} \in V$. Then, is $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ linearly independent? Yes, no, maybe?
   (b) Suppose $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ spans V. Show that $\{\mathbf{v}_1, \ldots, \mathbf{v}_n, \mathbf{v}_{n+1}\}$ also spans V.

5. Is the set of polynomials $\{x^2, 4x^2 - 2, 1\}$ linearly independent in $\mathbb{P}_2$? If not, find a subset of $\{x^2, 4x^2 - 2, 1\}$ which is linearly independent.

6. Suppose $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for V. Show that $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_3\}$ is also a basis for V.

7. Suppose V is a subspace of W. Show that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ are linearly independent in V, then they are linearly independent in W.

8. Listed here are the 8 axioms of a vector space:

1. $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$	(addition commutative)
2. $(\mathbf{v} + \mathbf{w}) + \mathbf{x} = \mathbf{v} + (\mathbf{w} + \mathbf{x})$	(addition associative)
3. $\exists\, \mathbf{0} \in V$ such that $\mathbf{v} + \mathbf{0} = \mathbf{v}$ for all $\mathbf{v}$	(additive identity)
4. $\forall\, \mathbf{v} \in V$, $\exists\, (-\mathbf{v}) \in V$ such that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$	(additive inverse)
5. $r(\mathbf{v} + \mathbf{w}) = r\mathbf{v} + r\mathbf{w}$	(distributive)
6. $(r + s)\mathbf{v} = r\mathbf{v} + s\mathbf{v}$	(distributive)
7. $r(s\mathbf{v}) = (rs)\mathbf{v}$	(scalar associative)
8. $1\mathbf{v} = \mathbf{v}$	(scalar identity)

   Using only vector space axioms, show the following properties of vector spaces (justify all your steps):
   (a) $\mathbf{v} + (\mathbf{0} + -\mathbf{v}) = \mathbf{0}$
   (b) If $\mathbf{v} + \mathbf{w} = \mathbf{0}$, then $\mathbf{w} = -\mathbf{v}$.

9. Let $S = \{ax^2 + bx + c \in \mathbb{P}_2 \mid a + b - 2c = 0\} \subset \mathbb{P}_2$.
   (a) Show S is a subspace of $\mathbb{P}_2$.
   (b) Find a basis for S.

10. (a) Is $\{(1, 2, 3), (0, 1, 7), (-1, 4, -8), (3, 0, 4)\}$ a set of linearly independent vectors in $\mathbb{R}^3$?
    (b) Does $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ span $\mathbb{R}^3$?