LB 118, Sections 009 & 010, Fall 2015
Homework 4 (due 10/09)

Instructions: Please write your solutions to the problems below on a clean piece of paper (not this piece of paper). You will not need more than one page (front and back) to write your answers. Show the steps taken to arrive at each answer. Do not include scratch work, doodles, scribbles, crossed out work, etc.; instead, carefully write your solutions after you have figured out the answers and checked them over.

You may work with other students on homework problems. For this assignment, each student must submit his or her own solution to the first problem. But, for the second problem, you may partner with up to three other students and submit one solution for your group; each student in the group will receive the same score for the second problem.

1. *As with previous homework assignments, this first problem is an exam problem from a previous semester of LB 118.* You do not need to simplify your answers.

   (a) Compute the derivative of $f(x) = \frac{1-3x^2+5x^4-7x^6}{x^3}$.

   (b) Compute the derivative of $g(x) = \frac{\sqrt{x+1}}{\sqrt{x-1}}$.

   (c) Compute the derivative of $h(x) = (\sin x)(\cos x)(\tan x)$.

   (d) Compute the derivative of $(1 + x + x^2)^{-3}$.

2. *As with previous homework assignments, this second problem is more challenging and is designed to strengthen your ability to extend ideas discussed in class and in the textbook to more complex situations.*

   The electrical potential V due to a "point charge" located at $x = a$ is given by the following function:

   $$V(x) = \frac{1}{4\pi\epsilon_0}\frac{Q}{|x-a|},$$

   where Q is a positive constant, measured in Coulombs, representing the magnitude of the charge and where ϵ_0 is a positive constant called the electric constant. Given x_0, the electrical potential, $V(x_0)$, represents the work required to move a unit positive electric change "from infinity" to $x = x_0$.

(a) Sketch a graph of $V(x)$. Do not choose values for any of the constants (a, Q, ϵ_0); instead leave these as unspecified constants. You may use a calculator or Wolfram Alpha or a similar graphing program to help you draw this sketch.

(b) Compute $V'(x)$. (Check your answer by comparing when $V'(x)$ is positive or negative to where a tangent line to the graph of $V(x)$ has positive or negative slope.)

(c) Let $U(x) = \frac{Q}{4\pi\epsilon_0}\left\{\frac{1}{|x-a|} + \frac{1}{|x+a|}\right\}$. The function U represents the electrical potential due to two point charges of equal charge but with one located at $x = a$ and the other located at $x = -a$. Sketch a graph of $U(x)$. As before, you may use a calculator or computer to help you sketch the graph.

(d) Compute $U'(x)$. (Check your answer by comparing when $U'(x)$ is positive, negative, or zero to where a tangent line to the graph of $U(x)$ has positive, negative, or zero slope.