

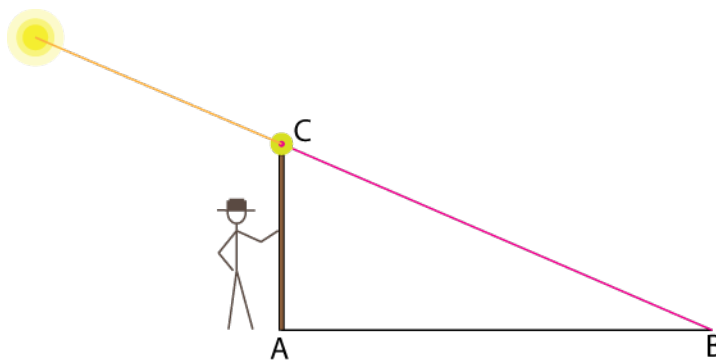
LB 118, Sections 009 & 010, Fall 2015
Homework 6 (due 10/21)

Instructions: Please write your solutions to the problems below on a clean piece of paper (not this piece of paper). You will not need more than one page (front and back) to write your answers. Show the steps taken to arrive at each answer. Do not include scratch work, doodles, scribbles, crossed out work, etc.; instead, carefully write your solutions after you have figured out the answers and checked them over.

You may work with other students on homework problems. For this assignment, each student must submit his or her own solution to the first problem. But, for the second problem, you may partner with up to three other students and submit one solution for your group; each student in the group will receive the same score for the second problem.

1. *As with previous homework assignments, this first problem is an exam problem from a previous semester of LB 118.*

Suppose that Dr. Jones is holding an 8 foot tall staff with a lens at the top of the staff. The base of the staff is at the point A and the lens is at the point C as in the figure below. Light from the sun is focused by the lens onto a point B on the floor. As the sun rises, the point B moves towards the archaeologist at a speed of 4 feet per minute. At what rate is angle $\angle ABC$ increasing when the length of AB is equal to $8\sqrt{3}$ feet. (Hint: the triangle is a 30-60-90 triangle.)



2. *As with previous homework assignments, this second problem is more challenging and is designed to strengthen your ability to extend ideas discussed in class and in the textbook to more complex situations.*

In this problem, you will derive a special case of the binomial theorem.

- (a) Let $f(x) = (1 + x)^n$, where n is a positive integer. Determine an expression for the k -th derivative of $f(x)$ for every positive integer k .
- (b) Suppose that $p(x)$ is the polynomial of degree n given by

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n,$$

where $a_0, a_1, a_2, \dots, a_n$ are constants. Compute the k -th derivative of $p(x)$ at $x = 0$.

- (c) If we expand $(1 + x)^n$, we obtain a polynomial $p(x)$ of degree n :

$$(1 + x)^n = p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n.$$

Use parts (a) and (b) to determine the values of the coefficients $a_0, a_1, a_2, \dots, a_n$. Hint: Evaluate the derivatives of both $(1 + x)^n$ and $p(x)$ at $x = 0$.