

**LB 118, Sections 009 & 010, Fall 2015**  
**Homework 7 & 8 (due 11/09)**

**Instructions:** Please write your solutions to the problems below on a clean piece of paper (not this piece of paper). Show the steps taken to arrive at each answer.

You may work with other students on homework problems. For this assignment, each student must submit his or her own solution to the first three problems. These first three problems are actual exam problems from previous semesters.

For the last problem, you may partner with up to three other students and submit one solution for your group; each student in the group will receive the same score for the last problem.

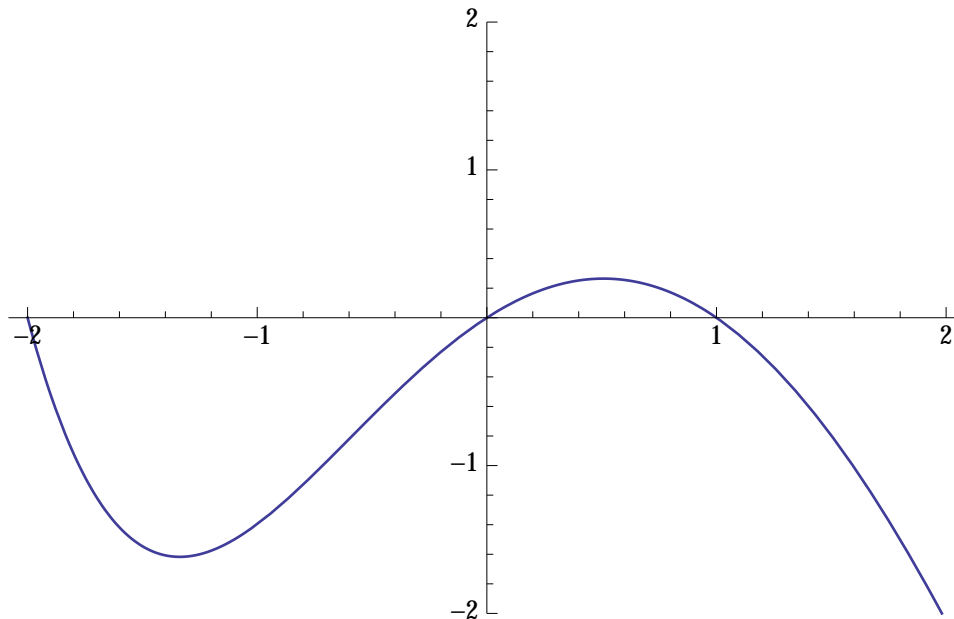
1. Determine the radius of the largest volume swimming pool having the shape of cylinder with a circular base and no top if the total surface area is limited to 2000 square feet. Justify that you have indeed found the solution which maximizes the volume.
2. The *derivative* of the function  $g(x)$  is given below.

$$g'(x) = \frac{(x + 1)}{(x - 2)(x + 2)^2}$$

Assume that the domain of  $g(x)$  is all real numbers except  $x = -2, 2$ . Use the above information to determine answers to each of the following questions:

- (a) On which intervals is  $g(x)$  increasing?
- (b) On which intervals is  $g(x)$  decreasing?
- (c) At which  $x$ -values does  $g(x)$  have a relative maximum?
- (d) At which  $x$ -values does  $g(x)$  have a relative minimum?

3. The graph of the *derivative* of the function  $f(x)$  is shown below.



Use this graph to determine answer to each of the following questions:

- On which intervals is  $f(x)$  concave up?
  - On which intervals is  $f(x)$  concave down?
  - At what  $x$ -coordinates are there points of inflection?
  - Sketch a graph of  $f(x)$ , assuming that  $f(0) = 0$ .
4. An important theorem which uses a limit at infinity is the Prime Number Theorem. Read the introduction, the “Statement,” and the “History” sections of the Wikipedia page on the Prime Number Theorem. Then answer the following questions:
- What are the first twenty prime numbers? (One is not a prime number.)
  - What is the statement of the Prime Number Theorem?
  - Use the conclusion of the Prime Number Theorem to estimate how many positive integers less than or equal to one trillion are prime numbers?
  - Use your previous answer to estimate the probability of choosing a prime when selecting a positive integer between one and one trillion at random.