Tutorial Worksheet, 03/01/2016

Instructions: Please work in groups of 3 or 4 students. Please work with students who will attend the same recitation section. You do not turn this worksheet in at the end of class; instead, attendance will be recorded so that you get credit for participating in this activity.

03/01: The Chain Rule

Derivative Rules. The following is a list of formulas for derivatives that we have studied thus far.

- (Linearity)
  1. $(F(x) + G(x))' = F'(x) + G'(x)$ and
  2. $(c \cdot F(x))' = c \cdot F'(x)$ for any constant c.
- (Power Rule)
  1. $(x^n)' = nx^{n-1}$ for any integer n.
  2. More generally, $((F(x))^n)' = n(F(x))^{n-1} \cdot F'(x)$.
- (Generalized Power Rule)
  1. $(x^\alpha)' = \alpha x^{\alpha-1}$ for any real number α.
  2. More generally, $((F(x))^\alpha)' = \alpha(F(x))^{\alpha-1} \cdot F'(x)$.

The following rules are special cases of the Generalized Power Rule, but they are encountered so frequently that they are worth singling out:

- (Reciprocal Rule)
  1. $(1/x)' = -1/x^2$
  2. More generally, $\left(\dfrac{1}{F(x)}\right)' = -\dfrac{1}{(F(x))^2} \cdot F'(x)$.
- (Square Root)
  1. $(\sqrt{x})' = 1/(2\sqrt{x})$
  2. More generally, $(\sqrt{F(x)})' = \frac{1}{2\sqrt{F(x)}} \cdot F'(x)$.

  In most of the above statements, part 2 says that "More generally..." we can use these rules when any function—not just the function $f(x) = x$—the is raised to a power. The most general statement to this effect is known as the *chain rule*:

$$(G(F(x)))' = G'(F(x)) \cdot F'(x).$$

Exercises.

1. Compute each of the following derivatives using the rules given on the previous page.

   (a) $(2x^{42} + 3^3 + 4x^{-2} - 3x^{-4})'$

   (b) $(2x^3 - 3/x)'$

   (c) $((x + 2x^3)^{10})'$

   (d) $(\sqrt{1+x})'$

   (e) $(\sqrt{1+x^2})'$

   (f) $\left(1 + \sqrt{x} - \sqrt{x}\right)'$

   (g) $\left(\dfrac{1}{x^{-2} + x + x^2}\right)'$

   (h) $\left(\dfrac{3}{2 + \sqrt{1+x}}\right)'$

   (i) $\left(\dfrac{3}{2x^3 + \sqrt{1+x+x^2}}\right)'$

2. Determine an equation of the line tangent to the given curve at the given point.

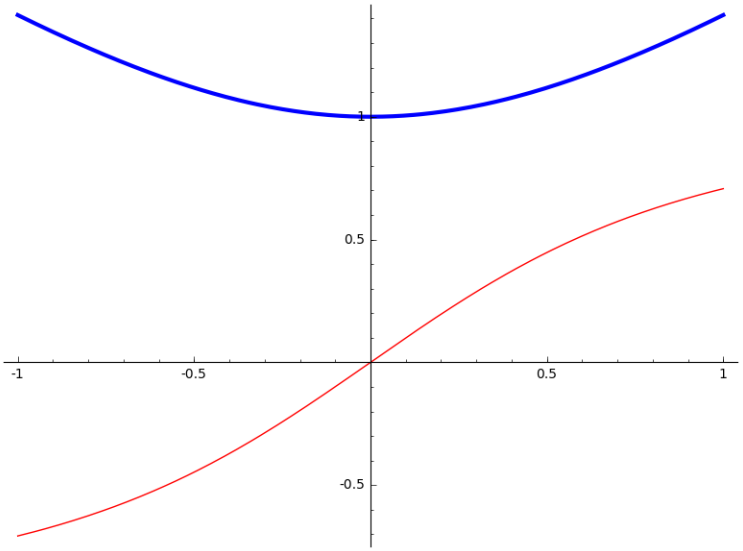
   (a) Curve: $y = x^3 - x$. Point: $(1, 0)$.

   (b) Curve: $y = \sqrt{5 + x^2}$. Point: $(2, 3)$.

   (c) Curve: $x^2 + y^2 = 1$. Point: $(1/\sqrt{2}, 1/\sqrt{2})$.

   (d) Curve: $x^2 + y^2 = 4$. Point: $(\sqrt{3}, -1)$. Careful! Sketch the curve and point before using algebraic methods to solve this problem.

3. Use the graph below to answer the questions which follow. One of the curves is the graph of $F(x)$. The other is the graph of $F'(x)$.



(a) Which curve is the graph of $F(x)$ and which is the graph of $F'(x)$?

(b) Estimate the derivative of $(F(x))^2$ at $x = 1$.

(c) Estimate the derivative of $\sqrt{F(x)}$ at $x = -1$.

(d) Estimate the derivative of $F(F(x))$ at $x = 0$.

(e) Estimate the derivative of $1/F(x)$ at $x = 1$.