

Tutorial Worksheet, 02/16/2016

Instructions: Please work in groups of 3 or 4 students. Please work with students who will attend the same recitation section. You do not turn this worksheet in at the end of class; instead, attendance will be recorded so that you get credit for participating in this activity.

02/16: Numerical, Graphical, and Algebraic Computation of Limits and Derivatives

Numerical Computations. Try to compute the following without using your calculator. You can use your calculator to check your work.

Let $F(x) = 10x^2$. Let $a = 1$. Make a table with five columns with the following headings: b , $b - a$, $F(b)$, $F(b) - F(a)$, and $(F(b) - F(a))/(b - a)$. The fill in the table using the following values of b : $b \in \{2, 1.1, 1.01\}$. Hint: To compute, for instance, $(1.01)^2$ by hand write it as $(1 + .01)^2$ and expand this using the binomial expansion $(x + y)^2 = x^2 + 2xy + y^2$ as shown below.

$$(1 + .01)^2 = 1^2 + 2(1)(.01) + (.01)^2$$

Use the table you created to estimate the slope of the tangent line to the graph of $F(x)$ at the point $(1, 10)$.

Graphical Computations.

Suppose that $F(x)$ is a function having the following values

x	1.5	1.9	2	2.1	2.5
$F(x)$	0.125	0.805	1.000	1.205	2.125

Estimate the rate of change of $F(x)$ at the point $(2, 1)$.

Algebraic Computations. There are three standard algebraic techniques for evaluating limits involving functions which only involve integer powers of x and roots of x .

- Factor, e.g. $x^2 - a^2 = (x - a)(x + a)$.
- Combine fractions, e.g. $\frac{1}{x} - \frac{1}{a} = \frac{a-x}{ax}$.
- Multiply by a conjugate, e.g. $(\sqrt{x} + \sqrt{a})(\sqrt{x} - \sqrt{a}) = x - a$, where $x, a \geq 0$.

Each of the problems below can be solved using one of the above techniques. In each case, you will need to simplify the algebraic expression so that the limit can be evaluated. Show all of your steps.

1. Compute $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$.

2. Compute $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$.

3. Compute $\lim_{b \rightarrow a} \frac{3b^2 - 3a^2}{b - a}$.

4. Let $F(x) = 7x^2$. Compute $F'(x)$ using the definition of the derivative:

$$\lim_{x \rightarrow a} \frac{F(x) - F(a)}{x - a}.$$

5. Let $F(x) = \frac{3}{x}$. Use the definition of the derivative to compute $F'(x)$. The answer is $-3x^{-2}$. Your objective is to show how this is obtained from the definition of the derivative.

6. Let $F(x) = 6\sqrt{3x}$. Use the definition of the derivative to compute $F'(x)$. The answer is $\frac{9}{\sqrt{3x}}$. Your objective is to show how this is obtained from the definition of the derivative.