Tutorial Worksheet, 01/19/2016

Instructions: Please work in groups of 3 or 4 students and solve each of the problems below. Your LA can help you. You do not turn these in at the end of class, but your LA will take attendance so that you get credit for participating. We will go over the solutions in the next class. You can spend up to 30 minutes on this worksheet. Afterwards, your LA will give you a quiz. This week's quiz is on graphing equations and plotting data. Working through the problems below will help to prepare you for the quiz.

Sine and Cosine Waves

A **sine wave** is a graph of an equation of the form

$$y = A\sin(\omega(x-\varphi)).$$

The following quantities are characteristics of this sine wave:

1. A is the **amplitude**.
2. ω is the **angular frequency**
3. The **period** is $\mathrm{T} = \frac{2\pi}{\omega}$.
4. φ is the **horizontal shift** or **phase shift**

A **cosine wave** is defined analogously: just replace sin with cos in the definition above..

The following equations,

$$\cos(x-\frac{\pi}{2}) = \sin x \tag{1}$$

$$\sin(x+\frac{\pi}{2}) = \cos x \tag{2}$$

imply that every sine wave is a cosine wave and vice versa. A vertical translation of a sine/cosine wave is also called a sine/cosine wave.

Review of Transformations:

1. The graph of $y = f(x-h)$ is the same as the graph of $y = f(x)$ translated horizontally to the right by h.
2. The graph of $y = f(x)+v$ is the same as the graph of $y = f(x)$ translated vertically upward by v.
3. The graph of $y = sf(x)$ is the same as the graph of $y = f(x)$ stretched (relative to the x-axis) vertically by a factor of s if $s > 0$.

4. The graph of $y = f(cx)$ is the same as the graph of $y = f(x)$ compressed (relative to the y-axis) horizontally by a factor of c if $c > 0$.

Notes:

- If $0 < s < 1$, the vertical stretch is more accurately described as a "vertical compression."
- If $0 < c < 1$, the horizontal compression is more accurately described as a "horizontal stretch."
- If $s < 0$, then the graph is reflected in the y-axis and stretched vertically by a factor of $|s|$.
- If $c < 0$, then the graph is reflected in the x-axis and compressed horizontally by a factor of $|c|$.

Order of Transformations:

The order in which a sequence of transformations is performed matters.

Exercise

1. Let $f(x) = \sin x$. Sketch the graph of $y = f(x)$. Make a careful sketch. You should clearly indicate the zeroes, i.e. the points on the graph where it touches the x-axis. You will use this same function in problems 2–4 below.

2. Sketch a graph of $y = f(x)$ that has, first, been translated to the right by $\pi/2$ and then, second, has its period compressed to π.

3. Sketch a graph of $y = f(x)$ that has, first, had its period compressed to π and then, second, has been translated to the right by $\pi/2$.

4. Consider the following two equations below. Which one did you graph in problem 2? Which one did you graph in problem 3?

   (a) $y = \sin(2(x - \frac{\pi}{2}))$

   (b) $y = \sin(2x - \frac{\pi}{2})$

From the previous exercise, we can conclude that to graph the equation $y = f(c(x - h))$ we should

- first, compress the graph of $y = f(x)$ horizontally by a factor of c (and so graph $y = f(cx)$) and then,
- second, translate the graph of $y = f(cx)$ to the right by h (and, thereby, graph $y = f(c(x - h)$, as desired).

Exercise: For each equation below, identify the amplitude, angular frequency, period, and phase shift. Then graph each equation. Use the technique above: write the wave in the form $y = \sin(\omega(x - \varphi))$ and, first, graph $y = \sin(\omega x)$ and then, second, translate this graph to the right by φ. (Use the analogous method in the case of a cosine wave.)

1. $y = \sin(2x + \pi)$

2. $y = 3\cos\left(\frac{x}{2}\right)$

3. $y = 10\sin\left(\frac{\pi}{10}x\right) + 3$

4. $y = \frac{1}{2}\cos\left(4x + \pi\right) - 1$

Exercise: Suppose that a buoy is bobbing up and down in a harbor. The buoy reaches a maximum elevation of 0.5 meters above sea level and a minimum elevation of 0.5 meters below sea level. The time it takes for the buoy to descend from its maximum elevation to its minimum elevation is 0.75 seconds. Write an equation which models the elevation of the buoy as a function of time.

Exercise: Find values of A, B, C, and D so that the graph of

$$y = A\cos\left(B(x - C) + D\right.$$

fits the data in Table 1.

Table 1: Average daily high temperature in Detroit

Month (1 = Jan.)	1	2	3	4	5	6	7	8	9	10	11	12
High (°F)	31	34	45	58	70	79	83	81	74	61	48	46

For additional practice on exercises like those on this worksheet, please sign up for Khan Academy as follows:

- Sign up at `khanacademy.org` (or log in with an existing account).
- Visit `khanacademy.org/coaches`.
- There, in the "Add a coach" field, enter the class code BYGKUR.