Tutorial Worksheet, 02/22/2016

Instructions: Please work in groups of 3 or 4 students. Please work with students who will attend the same recitation section. You do not turn this worksheet in at the end of class; instead, attendance will be recorded so that you get credit for participating in this activity.

02/22: Properties of Limits and Derivatives

Properties of Limits. The following properties hold for limits. Take a moment to read through this list.

1. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then
   (a) $\lim_{x\to a}(f(x) \pm g(x)) = L \pm M$.
   (b) $\lim_{x\to a} cf(x) = cL$, whenever c is a constant.
   (c) $\lim_{x\to a} f(x) \cdot g(x) = L \cdot M$.
   (d) $\lim_{x\to a} f(x)/g(x) = L/M$ provided $M \neq 0$.
2. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to L} g(x) = M$, then $\lim_{x\to a} g(f(x)) = M$.

Exercise. Write a few sentences explaining why the above properties should be true.

Properties of Derivatives. From the above properties of limits, one can deduce the following properties of derivatives. The properties of derivatives are used frequently in calculus. You should memorize these properties. In each of the statements below, it is assumed that $F(x)$ and $G(x)$ are differentiable, i.e. these functions have a well-defined tangent line at each point in their domains.

1. $(F(x) \pm G(x))' = F'(x) \pm G(x)$
2. $(cF(x))' = cF'(x)$, whenever c is a constant.
3. $(F(x) \cdot G(x))' = F'(x) \cdot G(x) + F(x) \cdot G'(x)$. This is called the *product rule*.
4. $(F(x)/G(x))' = (G(x) \cdot F'(x) - F(x) \cdot G'(x))/(G(x) \cdot G(x))$, whenever $G'(x) \neq 0$. This is called the *quotient rule*.
5. $(G(F(x)))' = G'(F(x)) \cdot F'(x)$. This is called the *chain rule*.

Exercise. Suppose that $f(x)$ and $g(x)$ are differentiable functions and that

- $f(0) = 1$, $f(1) = -2$, $f'(0) = 3$, $f'(1) = -1$, and
- $g(0) = 0$, $g(1) = 1/2$, $g'(0) = 7$, and $g'(1) = -5$.

Compute the value of each of the following:

1. the derivative of $f(x) + g(x)$ at $x = 1$, i.e. compute $(f + g)'(1)$.
2. $(f - g)'(0)$
3. the derivative of $f(x) \cdot g(x)$ at $x = 0$, i.e. compute $(fg)'(0)$.
4. $(fg)'(1)$.
5. $(ff)'(1)$.
6. the derivative of $f(x)/g(x)$ at $x = 1$, i.e. compute $(f/g)'(1)$.
7. $(g/f)'(1)$.
8. $(g/f)'(0)$.
9. the derivative of $f(g(x))$ at $x = 0$, i.e. compute $(f \circ g)'(0)$.
10. $(g \circ f)'(0)$.
11. $(f \circ f)'(0)$.
12. $(g \circ g)'(0)$.