Instructions: Please work through the exercises below. Pause at regular intervals and compare your answers to those of your neighbor. Help each other to find errors and correct them. You do not turn this worksheet in at the end of class; instead, attendance will be recorded so that you get credit for participating in this activity.

03/22: More Derivative Rules. Today we will practice computing the derivative of exponential & logarithmic functions. We will also use the product rule and the quotient rule.

Derivative Rules.

- (Linearity)

  1. $(F(x) + G(x))' = F'(x) + G'(x)$ and
  2. $(c \cdot F(x))' = c \cdot F'(x)$ for any constant c.

- (Power Rule)

  1. $(x^n)' = nx^{n-1}$ for any integer n.
  2. More generally, $((F(x))^n)' = n(F(x))^{n-1} \cdot F'(x)$.

- (Generalized Power Rule)

  1. $(x^\alpha)' = \alpha x^{\alpha-1}$ for any real number α.
  2. More generally, $((F(x))^\alpha)' = \alpha(F(x))^{\alpha-1} \cdot F'(x)$.

- (Reciprocal Rule: special case of x^{-1})

  1. $(1/x)' = -1/x^2$
  2. More generally, $\left(\dfrac{1}{F(x)}\right)' = -\dfrac{1}{(F(x))^2} \cdot F'(x)$.

- (Square Root: special case of $x^{1/2}$)

  1. $(\sqrt{x})' = 1/(2\sqrt{x})$
  2. More generally, $(\sqrt{F(x)})' = \dfrac{1}{2\sqrt{F(x)}} \cdot F'(x)$.

- (Exponential Functions)

  1. $(e^x)' = e^x$; note: e^x is sometimes written as $\exp(x)$.
  2. More generally, $(\exp(F(x)))' = \exp(F(x)) \cdot F'(x)$.

- (Logarithmic Functions)

  1. $(\ln|x|)' = 1/x$
  2. More generally, $(\ln|F(x)|)' = \dfrac{1}{F(x)} \cdot F'(x)$.

Exercises.

1. Compute each of the following derivatives using the rules given on the previous page. We have not yet practiced computing derivatives of logarithmic functions. Please ask your LA to work a few examples if you need help getting started.

   (a) $4x^2 - 7\sqrt{2 + x^3} + 5e^{-x}$

   (b) $\frac{5}{7} \cdot x^{-2/3} + 3\ln|x|$

   (c) $((x + e^x)^{10})'$

   (d) $(\sqrt{1 + \ln|x|})'$

   (e) $(\sqrt{1 + e^{2x}})'$

   (f) $\left(1 + \sqrt{e^x} - \sqrt[3]{e^x}\right)'$

   (g) $\left(\dfrac{1}{e^{-2x} + e^{2x}}\right)'$

   (h) $\exp(1 + x + x^2)$

   (i) $\ln|1 + x + x^2|$

2. Determine an equation of the line tangent to the given curve at the given point.

(a) Curve: $y = e^{3x} - x$. Point: $(0, 1)$.

(b) Curve: $y = \ln|1 + x^2|$. Point: $(1, \ln 2)$.

(c) Curve: $e^{2x} + \ln|y| = 1$. Point: $(0, 1)$.

The Product & Quotient Rules. In the time that remains, practice using the product & quotient rules.

- (Product Rule)

$$(F(x) \cdot G(x))' = F'(x) \cdot G(x) + F(x) \cdot G(x)$$

- (Quotient Rule)

$$\left(\frac{F(x)}{G(x)}\right)' = \frac{G(x) \cdot F'(x) - F(x) \cdot G'(x)}{G(x) \cdot G(x)}$$

Your LA can teach you a way to remember the quotient rule.

Exercise. Compute the derivative of each expression.

1. $x^2 \ln|x|$

2. $\sqrt{x + x^2} \cdot e^{2x}$

3. $(1 + x + x^2)^3 \cdot (1/x^2 - 1/x + 1)^5$

4. $\dfrac{x}{1 + x^2}$

5. $\dfrac{x^2}{1 - x^3}$