Tutorial Worksheet, 02/23/2016

Instructions: Please work in groups of 3 or 4 students. Please work with students who will attend the same recitation section. You do not turn this worksheet in at the end of class; instead, attendance will be recorded so that you get credit for participating in this activity.

02/23: Properties of Limits and Derivatives

Review: Computation of Limits and Rates of Change.

1. Compute $\lim_{x \to -1} \dfrac{(2x+1)^2 - 1}{x+1}$.

2. Suppose that $y = (2x+1)^2$. Write a limit which expresses the rate of change of y with respect to x at the point $(-1, 1)$. What is the value of this limit?

3. Suppose that $F(x) = 5 - 2x + x^2$ if $x \neq 0$ and that $F(x) = 3$ if $x = 0$. What is the limit of $F(x)$ as x approaches zero?

4. Suppose that $G(x) = x^2$ if $x > 0$ and $G(x) = x + 1$ if $x < 0$. What is the limit of $G(x)$ as x approaches zero?

Review: The Definition of the Derivative. The derivative of the function $F(x)$ is the function $F'(x)$ defined as follows:

$$F'(x) = \lim_{b \to x} \frac{F(b) - F(x)}{b - x}.$$

Exercise: Use the definition of the derivative to compute $F'(x)$ where $F(x) = \sqrt{x} - x^{-1}$. The answer is $F'(x) = \frac{1}{2\sqrt{x}} + x^{-2}$.

Hint: Keep the terms which come from the square root function separate from those which come from the function x^{-1}. You can separate a fraction as follows:

$$\frac{A + B}{C} = \frac{A}{C} + \frac{B}{C},$$

effectively "unadding" the two fractions.

If you want to try an easier exercise first, compute $G'(x)$ and $H'(x)$ where $G(x) = \sqrt{x}$ and $H(x) = -x^{-1}$.