LB 220, Sections 001 & 002, Fall 2015
Homework 5 (due 10/14)

Instructions: Please write your solutions to the problems below on a clean piece of paper (not this piece of paper). You will not need more than one page (front and back) to write your answers. Show the steps taken to arrive at each answer. Do not include scratch work, doodles, scribbles, crossed out work, etc.; instead, carefully write your solutions after you have figured out the answers and checked them over.

You may work with other students on homework problems. For this assignment, each student must submit his or her own solution to the first problem. But, for the second problem, you may partner with up to three other students and submit one solution for your group; each student in the group will receive the same score for the second problem.

1. *As with previous homework assignments, this first problem is an exam problem from a previous semester of LB 220.*

   In what sense is the partial derivative $f_x(a,b)$ of a function $f(x,y)$ at a point (a,b) a slope? Explain.

2. *As with previous homework assignments, this second problem is more challenging and is designed to strengthen your ability to extend ideas discussed in class and in the textbook to more complex situations.*

   Let $z = f(x,y)$ be a twice continously differentiable function of x and y. Let $x = r\cos\theta$ and $y = r\sin\theta$ be the equations which transform polar coordinates into rectangular coordinates. Use the chain rule to show that
   $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}.$$

   Note: It is very easy to make some fundamental errors. When computing the second derivatives, keep in mind that both f_x and f_y are also functions of x and y and, therefore, depend on r and θ. Because of this, you will need to apply the chain rule in situations where this might not be apparent. For example,
   $$(f_x \cos\theta)_r = (f_{xx}\cos\theta + f_{xy}\sin\theta)\cos\theta.$$