LB 220, Sections 001 & 002, Fall 2015
Homework 7 & 8 (due 11/09)

Instructions: Please write your solutions to the problems below on a clean piece of paper (not this piece of paper). Show the steps taken to arrive at each answer.

You may work with other students on homework problems. For this assignment, each student must submit his or her own solution to the first three problems. But, for the last problem, you may partner with up to three other students and submit one solution for your group; each student in the group will receive the same score for the last problem.

1. Sketch the region of integration and then compute the integral:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx.$$

2. Let R be the region in the first quadrant which lies between the circles centered at the origin having radii 1 and 3. Sketch this region and then compute the following integral:

$$\iint_R \sin\left(x^2 + y^2\right) dA.$$

3. Compute the average value of the function $f(x, y) = (1 + x^2 + y^2)^{-1}$ over the unit disk centered at the origin.

4. (a) Compute $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2}\, dx\, dy$ by converting to polar coordinates.

   (b) Explain how the following series of steps shows that the value of the integral in part (a) is also equal to $(\int_{-\infty}^{\infty} e^{-x^2}\, dx)^2$.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2}\, dx\, dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2} e^{-y^2}\, dx\, dy =$$

$$\int_{-\infty}^{\infty} e^{-y^2} \left(\int_{-\infty}^{\infty} e^{-x^2}\, dx \right) dy = \left(\int_{-\infty}^{\infty} e^{-x^2}\, dx \right) \left(\int_{-\infty}^{\infty} e^{-y^2}\, dy \right)$$

   (c) Finally, determine the value of $\int_{-\infty}^{\infty} e^{-x^2}\, dx$ by using parts (a) and (b).