309 Worksheet 3.3

True or False? Justify your answer:
Let V be a vector space.

(1) For all $\mathbf{v} \in V$ the set $\{\mathbf{v}\}$ is linearly independent.
True — False?
REASON:

(2) If $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \subseteq V$ with $\mathbf{v}_i = \mathbf{0}$ for some $1 \leq i \leq n$ then B is linearly dependent.
True — False?
REASON:

(3) The set of vectors $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \subseteq V$ is linearly dependent if and only if every vector $\mathbf{v}_i$ can be written as a linear combination of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_{i-1}, \mathbf{v}_{i+1}, \ldots, \mathbf{v}_n$.
True — False?
REASON:

(4) Three vectors $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$ are linearly dependent if and only if $\operatorname{span}(\mathbf{x}, \mathbf{y}, \mathbf{z})$ is a plane in $\mathbb{R}^3$.
True — False?
REASON:

(5) If $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \subseteq V$ is a linearly independent subset of V then every nonempty subset $B' \subseteq B$ is linearly independent.
True — False?
REASON:

(6) If $B = \{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ is a set of vectors in a vector space V so that for all $1 \leq i < j \leq n$ the set $\{\mathbf{u}_i, \mathbf{u}_j\}$ is linearly independent, then the set B is linearly independent.
True — False?
REASON: