309 Worksheet 3.4

True or False? Justify your answer:
Let V be a finite-dimensional vector space.

(1) If $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \subseteq V$ is linearly independent then every spanning set of V has at least n elements.
True — False?
REASON:

(2) Every vector space is finite dimensional.
True — False?
REASON:

(3) If a finite set of nonzero vectors B spans a vector space V, then some subset of B is a basis of V.
True — False?
REASON:

(4) If $\dim V \geq 1$ then V has infinitely many different bases.
True — False?
REASON:

(5) Let $S, T \subseteq V$ be subspaces of V with $V = S + T$, B_S a basis of S, and B_T a basis of T. Then $B_S \cup B_T$ is a basis of V.
True — False?
REASON:

(6) If a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ spans a finite dimensional vector space V and if T is a set of more than p vectors in V, then T is linearly dependent.
True — False?
REASON: