309 Worksheet 3.5

True or False? Justify your answer:
Let V be a finite-dimensional vector space.

(1) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \subseteq V$ is a linearly dependent set and $\mathbf{v}_{n+1} \notin \mathrm{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_n, \mathbf{v}_{n+1}\}$ is linearly independent.
True — False?
REASON:

(2) Let $S \subseteq V$ be a subspace. Then $S = V$ if and only if $\dim S = \dim V$.
True — False?
REASON:

(3) Suppose that $\dim V = n$. Then there is a chain of subspaces of V: $S_0 \subseteq S_1 \subseteq \ldots \subseteq S_{n-1} \subseteq S_n = V$ with $\dim S_i = i$.
True — False?
REASON:

(4) A vector space is infinite-dimensional if it is spanned by an infinite set.
True — False?
REASON:

(5) If $S, T \subseteq V$ are subspaces then $S = T$ if $\dim S = \dim T$.
True — False?
REASON:

(6) If every set of p vectors of V fails to span V then $\dim V > p$.
True — False?
REASON: