309 Worksheet 4.2

True or False? Justify your answer:

(1) If two vectors are orthogonal, they are linearly independent.
True — False?
REASON:

(2) If $\mathbf{x}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$, then $\mathbf{x}$ is orthogonal to every vector in $\text{span}(\mathbf{u}, \mathbf{v})$.
True — False?
REASON:

(3) The orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$ is a scalar multiple of $\mathbf{y}$.
True — False?
REASON:

(4) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis of W, then multiplying $\mathbf{v}_3$ by a scalar c gives a new orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, c\mathbf{v}_3\}$.
True — False?
REASON:

(5) The Gram-Schmidt process produces from a linearly independent set $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ an orthogonal set $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ with the property that for every $1 \leq k \leq n$, the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ span the same subspace as $\mathbf{x}_1, \ldots, \mathbf{x}_k$.
True — False?
REASON:

(6) The set of all vectors in $\mathbb{R}^n$ which are orthogonal to one fixed vector is a subspace of $\mathbb{R}^n$.
True — False?
REASON: