309 Worksheet 6.2

True or False? Justify your answer:
Let V, W be vector spaces, $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ a basis of V.

(1) If $\mathbf{w}_1, \ldots, \mathbf{w}_n \in W$ are vectors in W then there is exactly one linear transformation $T : V \longrightarrow W$ with $T(\mathbf{v}_i) = \mathbf{w}_i$ for all $1 \leq i \leq n$.
True — False?
REASON:

(2) If $\mathbf{v}'_1, \ldots, \mathbf{v}'_m \in V$ are vectors with $\text{span}\{\mathbf{v}'_1, \ldots, \mathbf{v}'_m\} = V$ and $\mathbf{w}'_1, \ldots, \mathbf{w}'_m \in W$ vectors in W then there is a linear transformation $T : V \longrightarrow W$ with $T(\mathbf{v}'_i) = \mathbf{w}'_i$ for all $1 \leq i \leq m$.
True — False?
REASON:

(3) There are onto linear transformations $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$.
True — False?
REASON:

(4) There are one-to-one linear transformations $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$.
True — False?
REASON:

(5) There are onto linear transformations $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$.
True — False?
REASON:

(6) There are one-to-one linear transformations $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$.
True — False?
REASON: