Math 451: Numerical Analysis I

http://www.math.msu.edu/~seal/teaching/fa13/

Fall 2013

Michigan State University[a]

MWF 09:10–10:00 Wells Hall C304 (updated!)

Instructor: David Seal (seal@math.msu.edu)

[a]Note: there are many hyperlinks hidden in this document!

1 Contact and Meeting Information

1.1 Meeting times

Course meets three days a week (**MWF**) at **09:10–10:00** in **Wells Hall C304** (updated!).

It is your responsibility to learn the material presented in lecture. If you must miss a lecture for any reason, find notes from a classmate in order to make up the missing material.

1.2 Office Hours

I live in **Wells Hall C330**. I guarantee my presence there every **Monday** and **Wednesday** from **10:10–11:00**. Meetings outside of this time can be scheduled by appointment with appropriate notice and availability.

2 Course Overview

Math 451 is the first course in a two-semester sequence designed for undergraduate and graduate students working on scientific, engineering, statistics, and mathematics majors. This course serves as an introduction to numerical methods used to applied mathematics problems, with applications across the spectrum of disciplines.

2.1 Pre-requisites

Please visit the Registrar's website (https://www.reg.msu.edu) for a formal definition of what's required to enroll in this course. Their suggestions include a course on Linear Algebra, MTH 309/314/415 or equivalent; a course on differential equation, MTH 235/255/340 or equivalent; and an introductory programming course, CS 131/231 or equivalent.

2.1.1 Pre-requisites: Linear Algebra

You should have a working knowledge of basic Linear Algebra (e.g. MTH 309, 314 or 415) in order to enroll in this course. An *incomplete* list of material that you should be familiar with includes the following:

1. Matrix and vector operations (e.g. multiplication and addition).
2. Finding Eigenvalues and Eigenvectors for small linear systems.
3. Linear independence, and basis vectors.

It will be assumed that you've at least heard these words before.

2.1.2 Pre-requisites: Differential equations

Although the formal requirements ask for a working knowledge of differential equations (e.g. MTH 235, 255 or 340). In those courses, you will have been restricted to finding analytical solutions to a very small class of problems. For example topics that you should have heard include,

1. Separation of variables.
2. Solutions to linear differential equations.

2.1.3 Pre-requisites: A programming course?

Although the formal requirements ask for you having taken an introductory course on computer programming, (e.g. CS 131, 132, ...). This is not a hard requirement to enroll in the course. However, you will be asked to implement the techniques we investigate in a programming language of your choice, for example, Python, Matlab, C/C++ or Fortran are all acceptible.

Sample programs will be provided throughout the duration of the course, and therefore, a *willingness to learn a language* is the only hard pre-requisite required. As part of this course, we will discuss good programming skills.

2.2 Goals

The purpose of this course is to provide an introduction to numerical methods for solving applied mathematical models. As a broad subject area, Numerical Analysis sits within the interface of Mathematics, Computer Science (Engineering) and Applied Sciences (physics, biology, chemistry, electrical engineering, ...). We will seek to accomplish the following three goals with every problem introduced:

1. Mathematically formulate scientific problems.
2. Construct solutions to these problems using numerical methods.
3. Implement and interpret the numerical results using a programming language.

2.2.1 Textbook

Required textbook:

- Brian Bradie, *A Friendly Introduction to Numerical Analysis.*

Available on reserve in the library, from the MSU Bookstore, or from a private seller such as Schuler Books or Amazon.

Recommended textbooks:

- W. Cheney and D. Kincaid, *Numerical Mathematics and Computing.*
- K.E. Atkinson, *An Introduction to Numerical Analysis*

2.2.2 Course plan

We will cover the following Chapters from Bradie's book:

- Chapter 1: Algorithms, convergence (complexity), finite precision of floating point arithmetic.
- Chapter 2: Rootfinding: the bisection method, fixed point iteration, Newton's method and secant method. Finding roots of polynomials.
- Chapter 3: Linear systems of equations: Gaussian elimination with partial pivoting, LU decomposition, iterative techniques.
- Chapter 4: Eigenvalues and eigenvectors: the power method, inverse method, reduction to tridiagonal form.

- Chapter 5: Interpolation (and curve fitting): Lagrange and Newton form of interpolating polynomial, Piecewise linear interpolation, cubic spline interpolation.
- Chapter 6: Differentiation and Integration: Taylor series (review) Numerical differentiation, Richardson extrapolation, Newton-Cotes quadrature, Gaussian quadrature, adaptive quadrature.
- Chapter 7: Initial value problems of ordinary differential equations: Euler's method, Taylor methods, Runge-Kutta methods, Multistep methods, Convergence and Stability analysis.

3 Important Dates

Every day is important, some more than others. For a list of the more important dates, consider the following:

Aug 28	classes begin.
Sep 2	University holiday.
Oct 7	Exam 1.
Oct 14	One page project abstract due; Include group members, references.
Oct 16	Drop deadline.
Nov 4-7	20 minute meeting, group project status update.
Nov 25	Exam 2.
Nov 28, 29	University holiday.
Dec 2-6	Project presentations.
Dec 9	Project reports due by noon.

4 Evaluation

End of semester grades will be based on homework (35%), in class midterms (15% each), and a final project (35%). For each of the two exams, you will be permitted one sheet of notes, but no calculators.

4.1 Final Project

As part of your evaluation, you will be asked to produce results for a project of your choosing. If desired, you may work in a group of size up to 3. Suggestions will be provided upon request.

5 Keys to Success

Mathematics is the art of problem solving, and therefore in order to master this, you have to *solve problems*. Feel free to work out additional exercises from the textbook and other texts!

- Attend **Lecture**. Take complete notes in class: any supplemental material will be presented in lecture.
- Visit **Office Hours**.
- Read the textbook.
- Do *all* of the assigned **homework**. Start working on the assigned problems as soon as they're assigned.
- Take the **midterms**!

6 Academic Honesty and Integrity

Don't cheat.[1] Unacceptable behavior will not be tolerated and will be reported. You *do not* want to jeopardize all of the hard work you've put into your degree!

[1] Visit MSU's policy on "Student's Rights and Responsibilities" or "Graduate Students Rights and Responsibilities" for a full discussion.