5.4 Introduction to Factoring

A. Factoring out GCF

Factoring is “undoing” the distributive property/multiplication

$$3x(4 - y) = 12x - 3xy$$

distributive property

factoring

We can often factor by “stripping out” the largest factor common to all terms, and then writing what’s left inside parentheses.

B. Examples

Example 1: Factor $6x^2 - 8x$. Remove the GCF.

Solution

$6x^2$ and $-8x$ both have 2 and x in them.

Thus $2x$ is the largest factor, the GCF.

Thus write $2x(\quad)$, where we will write what is left on the inside.

Thus we have $2x(3x - 4)$ (remove $2x$ from $6x^2$ to get $3x$; similarly for $-8x$)

Ans $\boxed{2x(3x - 4)}$

Note: You can always check a factoring problem by remultiplying!

Example 2: Factor $18a^3b^2c - 30a^2bc^2 + 42a^2bc$. Remove the GCF.

Solution

$18a^3bc$, $-30a^2bc^2$, and $42a^2bc$ all have $6a^2bc$ in them.

Thus $6a^2bc$ is the largest factor, the GCF.

Thus write $6a^2bc(\qquad\qquad)$, where we will write what is left on the inside.

Ans $\boxed{6a^2bc(3ab - 5c + 7)}$

Example 3: Factor $7x(4a - 2) - 3(4a - 2)$. Remove the GCF.

Solution

$7x(4a - 2)$ and $-3(4a - 2)$ both have "$(4a - 2)$" in them.

Thus write $(4a - 2)(\qquad)$, where we will write what is left on the inside.

Ans $\boxed{(4a - 2)(7x - 3)}$

C. Factoring By Grouping

If you can't factor out the GCF, and the polynomial has four terms, we can **sometimes** factor further by **grouping in pairs**.

We factor out of just the first two, and just the last two. If we can **not** factor **even further**, then factoring by grouping **fails**.

As we'll see, this means that the **factor pairs must MATCH**.

D. Examples

Example 1: Factor $18ax - 24x - 15a + 20$ **if possible** by grouping the first and last pairs.

Solution

Note that there is no GCF here.

$$\underbrace{18ax - 24x}_{6x}\underbrace{-15a + 20}_{-5}$$

Thus write $6x(\quad) - 5(\quad)$, where we will fill in what's left.

Then we have $6x(3a - 4) - 5(3a - 4)$.

Notice that the FACTOR PAIRS $(3a - 4)$ and $(3a - 4)$ MATCH
(if they didn't match, factoring by grouping FAILS)

Now $(3a - 4)$ is common, so factor it out.

Ans $\boxed{(3a - 4)(6x - 5)}$

Example 2: Factor $18ax - 24x - 10a - 15$ **if possible** by grouping the first and last pairs.

Solution

Note that there is no GCF here.

$$\underbrace{18ax - 24x}_{6x}\,\underbrace{-10a - 15}_{-5}$$

Thus write $6x(\qquad) - 5(\qquad)$, where we will fill in what's left.

Then we have $6x(3a - 4) - 5(2a + 3)$.

Notice that the FACTOR PAIRS $(3a - 4)$ and $(2a + 3)$ DON'T MATCH.

Thus the method FAILS.

Ans $\boxed{18ax - 24x - 10a - 15 \text{ (not factorable)}}$

Note: As in the past two examples, whenever we factor with four terms, and the third term is negative, we factor out the $-$ from the last pair.

E. Factoring Four Terms–General Strategy

1. First factor out the GCF.

2. Then, given what remains, try factoring by grouping.

3. If factoring by grouping fails, try rearranging terms and try again
(i.e. try different pairs!)

Note: We sometimes use $[\quad]$ (brackets) when dealing with a lot of embedded parentheses.

F. Examples

Example 1: Factor $36a^2x^2 - 30ax - 54ax^2 + 45x$ completely.

Solution

1. $3x$ is common to all terms; factor it out

$$3x(12a^2x - 10a - 18ax + 15)$$

Now we will try factoring by grouping on the inside. Change to brackets due to embedded parentheses.

2. $3x[\underbrace{12a^2x - 10a}_{2a}\underbrace{-18ax + 15}_{-3}]$

$3x[2a(\qquad) - 3(\qquad)]$ fill in

$3x[2a(6ax - 5) - 3(6ax - 5)]$ factor pairs MATCH; factor it out

$3x[(6ax - 5)(2a - 3)]$

Outer brackets are now unnecessary (by associativity)

Ans $\boxed{3x(6ax - 5)(2a - 3)}$

Example 2: Factor $6x^4y^2 - 36xy^5 + 9x^2y^4 - 24x^3y^3$ completely.

Solution

1. $3xy^2$ is common to all terms; factor it out

$$3xy^2(2x^3 - 12y^3 + 3xy^2 - 8x^2y)$$

Now we will try factoring by grouping on the inside. Change to brackets due to embedded parentheses.

2. $$3xy^2[\underbrace{2x^3 - 12y^3}_{2}\underbrace{+3xy^2 - 8x^2y}_{xy}]$$

$$3xy^2[2(x^3 - 6y^3) + xy(3y - 8x)]$$

Factor pairs **don't** match. Factoring FAILS.

Try a different rearrangement.

3. $$3xy^2[2x^3 - 12y^3 + 3xy^2 - 8x^2y]$$

$$3xy^2[\underbrace{2x^3 - 8x^2y}_{2x^2}\underbrace{+3xy^2 - 12y^3}_{3y^2}]$$

$$3xy^2[2x^2(x - 4y) + 3y^2(x - 4y)]$$ works!

$$3xy^2[(x - 4y)(2x^2 + 3y^2)]$$

Outer brackets are now unnecessary (by associativity)

Ans $\boxed{3xy^2(x - 4y)(2x^2 + 3y^2)}$

G. Negative Factor Pairs

Negative factor pairs are pairs that look like this:

$3x - 4$ and $4 - 3x$

$2 - 5x$ and $5x - 2$

i.e. they are differences with the order reversed.

Basic Fact: Multiplying one of the negative factor pairs by -1 yields the other one.

Example: Given $2x - 5$, then $-1 \cdot (2x - 5) = -2x + 5 = 5 - 2x$

H. Negative Factor Pairs in Factoring

If you factor by grouping and get negative factor pairs, you can factor an additional minus sign out of one pair to get the pairs to match.

Example: Factor $ax^2 - 3a^2x - 6a + 2x$

Solution

NO GCF!

Try factoring by grouping as is:

$$ax(x - 3a) - 2(3a - x)$$

Factor pairs don't match, but are negative factor pairs . . .

Change $3a - x$ into $x - 3a$ via additional minus sign; that is, change the sign of the second term.

$$ax(x - 3a) + 2(x - 3a)$$

Now the factor pairs match; factor it out

Ans $\boxed{(x - 3a)(ax + 2)}$