5.7B Generalized Factoring II

A. More on Differences of Squares

Some difference of squares problems are trickier. Be careful with minus signs!

Example 1: Factor $(2x+3)^2 - 16y^2$ completely.

Solution

Two terms: Difference of Squares

$$(2x+3)^2 - (4y)^2$$

$$[(2x+3)+4y][(2x+3)-4y]$$

Ans $\boxed{(2x+3+4y)(2x+3-4y)}$

Example 2: Factor $25x^2 - (3y-2)^2$ completely.

Solution

Two terms: Difference of Squares

$$(5x)^2 - (3y-2)^2$$

$$[5x+(3y-2)][5x-(3y-2)]$$

Be careful with parentheses!

Ans $\boxed{(5x+3y-2)(5x-3y+2)}$

Example 3: Factor $(3x+5)^2 - (7x-1)^2$ completely.

Solution

Two terms: Difference of Squares

$$(3x+5)^2 - (7x-1)^2$$

$$[(3x+5)+(7x-1)][(3x+5)-(7x-1)]$$

$$[3x+5+7x-1][3x+5-7x+1]$$

$$(10x+4)(-4x+6)$$

Each factor has a GCF, so we have $[2(5x+2)][-2(2x-3)]$

Ans $\boxed{-4(5x+2)(2x-3)}$

B. Grouping Triplets and Perfect Squares

Sometimes factoring by grouping for four or more terms does not work with **any** rearrangement if you group in pairs. However, if you group three terms, you may be able to use perfect square factoring to turn the problem into a "complicated" difference of squares.

Example 1: Factor $4x^2 - 12xy + 9y^2 - m^2$ completely.

Solution

Factor by grouping: Group triplet (since m^2 doesn't belong)

$$(4x^2 - 12xy + 9y^2) - m^2$$

Perfect Square: $(2x-3y)^2 - m^2$

Difference of Squares!

$$[(2x-3y)+m][(2x-3y)-m]$$

Ans $\boxed{(2x-3y+m)(2x-3y-m)}$

Example 2: Factor $16x^2 - 9m^2 + 42m - 49$ completely.

Solution

$16x^2$ doesn't belong.

Group the last three, **but** when you group, $-$ distributes!

$$16x^2 - (9m^2 - 42m + 49)$$

Perfect Square: $16x^2 - (3m-7)^2$

Difference of Squares:

$$(4x)^2 - (3m-7)^2$$

$$[4x + (3m-7)][4x - (3m-7)]$$

Ans $\boxed{(4x+3m-7)(4x-3m+7)}$

Example 3: Factor $25m^2 - 16x^2 + 40xy - 25y^2$ completely.

Solution

$25m^2$ doesn't belong.

Group the last three, **but** when you group, $-$ distributes!

$25m^2 - (16x^2 - 40xy + 25y^2)$

Perfect Square: $25m^2 - (4x - 5y)^2$

Difference of Squares:

$(5m)^2 - (4x - 5y)^2$

$[5m + (4x - 5y)][5m - (4x - 5y)]$

Ans $\boxed{(5m + 4x - 5y)(5m - 4x + 5y)}$

C. More on Factoring By Grouping

When factoring by grouping, see if one term looks "different".

If there is one, try grouping by "triplets"; otherwise pairs.

Remember, you may need to rearrange terms to get it to work.

As always, check for a GCF first!

Example 1: Factor $m^3 - mn^2 - n^3 + nm^2$ completely.

Solution

NO GCF!

Everything looks the same, try pairs:

$$m(m^2 - n^2) - n(n^2 - m^2)$$

Negative factor pairs

$$m(m^2 - n^2) + n(m^2 - n^2)$$

$$(m^2 - n^2)(m + n)$$

Difference of squares

$$(m + n)(m - n)(m + n)$$

Switch the order

$$(m + n)(m + n)(m - n)$$

Ans $\boxed{(m + n)^2(m - n)}$

Example 2: Factor $4x^2 - 4k^2 + 25y^2 - 20xy$ completely.

Solution

NO GCF!

Since $-4k^2$ looks "different", we group in triplets.

Since $-4k^2$ is negative, we put it last:

$$(4x^2 + 25y^2 - 20xy) - 4k^2$$

Rearrange:

$$(4x^2 - 20xy + 25y^2) - 4k^2$$

Perfect Square:

$$(2x - 5y)^2 - 4k^2$$

Difference of Squares:

$$(2x - 5y)^2 - (2k)^2$$

$$[(2x - 5y) + 2k][(2x - 5y) - 2k]$$

Ans $\boxed{(2x - 5y + 2k)(2x - 5y - 2k)}$

Example 3: Factor $6ax^3 - 8a^2x^2 - 12a^2x + 7ax^2 - 3ax$ completely.

Solution

GCF: $ax(6x^2 - 8ax - 12a + 7x - 3)$

Rearrange:

$$ax[6x^2 + 7x - 3 - 8ax - 12a]$$

Group first three and last two:

$$ax[(6x^2 + 7x - 3) - 4a(2x + 3)]$$

Need to do AntiFOIL on $6x^2 + 7x - 3$:

$6x^2 + 7x - 3$	$\boxed{-18}$	TSP: $+, -$
$6x^2 + 8x - 1x - 3$	-8	
$6x^2 + 9x - 2x - 3$	-18 $\surd$	

$$3x(2x + 3) - 1(2x + 3)$$

$$(2x + 3)(3x - 1)$$

Thus, we have

$$ax[(2x + 3)(3x - 1) - 4a(2x + 3)]$$

Inside has a GCF of $(2x + 3)$ . . . factor it out:

$$ax\,[(2x + 3)\,((3x - 1) - 4a)]$$

Ans $\boxed{ax(2x + 3)(3x - 1 - 4a)}$
