5.8 Polynomial Equations Solvable By Factoring

A. Zero Product Principle

The zero product principle says that if a product of factors is zero, then one of the factors must be zero.

Example 1: Solve the equation $(x+3)(2x-1)=0$ for x.

Solution

By the zero product principle,

$$x+3=0 \quad \text{OR} \quad 2x-1=0$$

Thus, $x=-3 \quad \text{OR} \quad x=\frac{1}{2}$

Ans $\boxed{x=-3 \quad \text{OR} \quad x=\frac{1}{2}}$

Example 2: Solve the equation $x(3x-1)(2x+3)(x-3)=0$ for x.

Solution

By the zero product principle,

$$x=0 \quad \text{OR} \quad 3x-1=0 \quad \text{OR} \quad 2x+3=0 \quad \text{OR} \quad x-3=0$$

Thus, $x=0 \quad \text{OR} \quad x=\frac{1}{3} \quad \text{OR} \quad x=-\frac{3}{2} \quad \text{OR} \quad x=3$

Ans $\boxed{x=0 \quad \text{OR} \quad x=\frac{1}{3} \quad \text{OR} \quad x=-\frac{3}{2} \quad \text{OR} \quad x=3}$

B. Higher Order Equations

An equation of degree 2 is called a **quadratic equation**.

i.e. $3x^2 - 2x + 4 = 0$

An equation of higher degree is called a **polynomial equation**.

C. Solving Quadratic Equations and Polynomial Equations

Some (not all) quadratic equations can be solved by factoring.

Strategy

1. Move everything to one side of the equation, so the other side is zero.
2. Factor the polynomial.
3. Use the zero product principle.

D. Examples

Example 1: Solve $x^2 + 5x + 6 = 0$ for x.

Solution

1. The right hand side is already zero!

2. Factor the left hand side:

$$\begin{array}{c|ll}
x^2+5x+6=0 & \boxed{6} & \text{TSP: } +,+ \\
\hline
x^2+x+4x+6=0 & 4 & \\
x^2+2x+3x+6=0 & 6 \checkmark & \\
x(x+2)+3(x+2)=0 & & \\
(x+2)(x+3)=0 & &
\end{array}$$

3. Zero Product Principle:

$$x+2=0 \quad \text{OR} \quad x+3=0$$

Ans $\boxed{x=-2 \quad \text{OR} \quad x=-3}$

Example 2: Solve $2x^2-5x=12$ for x.

Solution

1. Move everything to one side: $2x^2-5x-12=0$

2. Factor the left hand side:

$$\begin{array}{c|ll}
2x^2-5x-12=0 & \boxed{-24} & \text{TSP: } +,- \\
\hline
2x^2+x-6x-12=0 & -6 & \\
2x^2+2x-7x-12=0 & -14 & \\
2x^2+3x-8x-12=0 & -24 \checkmark & \\
x(2x+3)-4(2x+3)=0 & & \\
(2x+3)(x-4)=0 & &
\end{array}$$

3. Zero Product Principle:

$$2x+3=0 \quad \text{OR} \quad x-4=0$$

Ans $\boxed{x=-\frac{3}{2} \quad \text{OR} \quad x=4}$

Example 3: Solve $(x-1)(x-2)=2$ for x.

Solution

We can't use the zero product principle now! The right hand side is **not** zero.

You have to multiply the left side out, move everything to the left, and then refactor:

$$x^2 - 2x - x + 2 = 2$$

$$x^2 - 3x + 2 = 2$$

$$x^2 - 3x = 0$$

Refactor: $x(x-3) = 0$

Zero Product Principle: $x = 0$ OR $x = 3$

Ans $\boxed{x = 0 \quad \text{OR} \quad x = 3}$

Example 4: Solve $x^3 + x^2 = 20x$ for x.

Solution

Move everything to one side: $x^3 + x^2 - 20x = 0$

Factor the left hand side: $x(x^2 + x - 20) = 0$

Now using AntiFOIL:

$$\begin{array}{c|l} x^2+x-20 & \boxed{-20} \quad \text{TSP: } +, - \\ \hline x^2+2x-x-20 & -2 \\ x^2+3x-2x-20 & -6 \\ x^2+4x-3x-20 & -12 \\ x^2+5x-4x-20 & -20 \; \surd \\ x(x+5)-4(x+5) & \\ (x+5)(x-4) & \end{array}$$

Thus, we have $x(x+5)(x-4)=0$

By the Zero Product Principle:

$$x=0 \quad \text{OR} \quad x+5=0 \quad \text{OR} \quad x-4=0$$

Ans $\boxed{x=0 \quad \text{OR} \quad x=-5 \quad \text{OR} \quad x=4}$

Example 5: Solve $x^3-3x^2-4x+12=0$ for x.

Solution

Factor by grouping: $x^2(x-3)-4(x-3)=0$

Thus we have $(x-3)(x^2-4)=0$

Difference of squares: $(x-3)(x+2)(x-2)=0$

Zero Product Principle: $x-3=0$ OR $x+2=0$ OR $x-2=0$

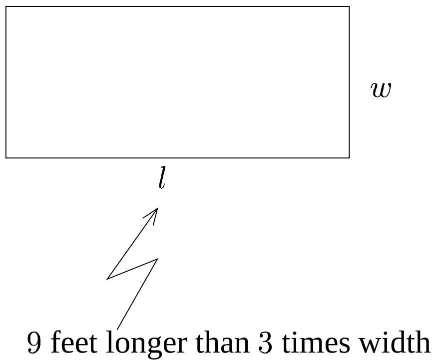
Ans $\boxed{x=3 \quad \text{OR} \quad x=-2 \quad \text{OR} \quad x=2}$

E. Solving Applied Problems Involving Polynomial Equations

Example: The length of a rectangle is 9 feet longer than 3 times the width. If the area is 210 square feet, find the rectangle's dimensions.

Solution

Picture



Thus $l = 3w + 9$

Now $A = lw$, so $A = (3w + 9)w$

Thus $(3w + 9)w = 210$

Then

$$3w^2 + 9w = 210$$

$$3w^2 + 9w - 210 = 0$$

$$3(w^2 + 3w - 70) = 0$$

Now factor $w^2 + 3w - 70$ by AntiFOIL:

$w^2 + 3w - 70$	$\boxed{-70}$	TSP: $+, -$
$w^2 + 4w - w - 70$	-4	
$w^2 + 5w - 2w - 70$	-10	
jump ahead		
$w^2 + 10w - 7w - 70$	-70 $\checkmark$	
$w(w + 10) - 7(w + 10)$		
$(w + 10)(w - 7)$		

Thus we have $3(w + 10)(w - 7) = 0$

By the Zero Product Principle: $3 = 0$ OR $w + 10 = 0$ OR $w - 7 = 0$

Since $3 = 0$ is nonsense, we have $w = -10$ OR $w = 7$.

Since we have a physical problem, we can't have $w = -10$!

Thus $w = 7$.

Since $l = 3w + 9$, $l = 3(7) + 9 = 21 + 9 = 30$

Ans $\boxed{\text{width is 7 feet and length is 30 feet}}$