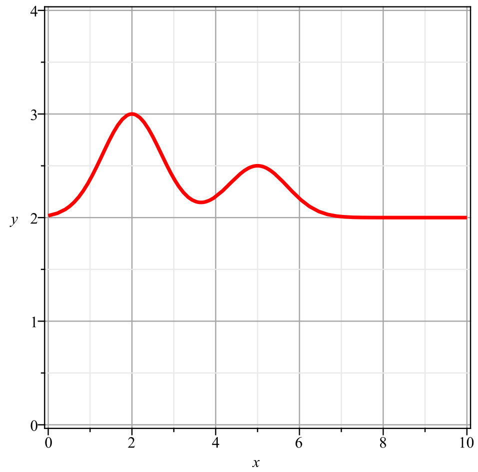
Review 1

MTH132-040, Calculus I

(1) The graph below describes the population of fruit flies (measured in hundreds) as a function of time, over a period of 10 days.



(a) Over what period of time is the instantaneous rate of change of the population negative?
Answer: $t \in (2, 3.5) \cup (5, 7)$

(b) Over what period of time is the instantaneous rate of change of the population positive?
Answer: $t \in (0, 2) \cup (3.5, 5)$

(c) On approximately what day is the derivative of the function, giving the population as a function of time, the greatest?
Answer: On approximately day 1, or a little after.

(d) Calculate the average rate of change of the population between day 2 and day 5.
Answer: $-\frac{1}{6}$

(2) Calculate the following limits.

(a) $\displaystyle\lim_{x\to 3} \frac{3x^2 - 5x + \pi}{x^2 - 3}$ (b) $\displaystyle\lim_{x\to 2^-} \frac{x+5}{x^2 - 3x + 2}$ (c) $\displaystyle\lim_{h\to 0} \frac{\frac{1}{x+5+h} - \frac{1}{x+5}}{h}$

(d) $\displaystyle\lim_{x\to -3} \frac{2 - \sqrt{x^2 - 5}}{x+3} + 3x - 1$ (e) $\displaystyle\lim_{x\to\infty} \frac{5x^3 - 7x + 1}{1 - x^4}$ (f) $\displaystyle\lim_{x\to -\infty} \frac{|x+5|}{x-4}$

(g) $\displaystyle\lim_{t\to 0} \frac{7\tan(3t)}{5t}$

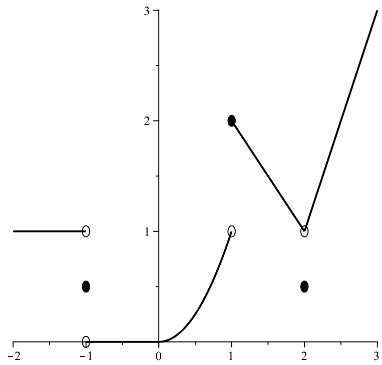
(h) Suppose $\displaystyle\lim_{s\to 3} f(s) = -1$ and $\displaystyle\lim_{s\to 3} g(s) = 6$, find $\displaystyle\lim_{s\to 3} \left(f^2(s) + 5f(s)g(s)\right)$.

Answer: (a) $\frac{12+\pi}{6}$, (b) $-\infty$, (c) $\frac{-1}{(x+5)^2}$, (d) $-\frac{17}{2}$, (e) 0, (f) -1, (g) $\frac{21}{5}$

(3) Use the Sandwich Theorem to find $\displaystyle\lim_{p\to 5} S(p)$, provided that $\displaystyle\frac{6-p}{p-2} \leq S(p) \leq \frac{\sin(p-5)}{3p-15}$.
Answer: $\frac{1}{3}$

(4) Using the graph of the function $y = f(x)$ given below, evaluate the following:

$$\lim_{x\to -1} f(x) = DNE$$
$$f(-1) = 0.5$$
$$\lim_{x\to 1^-} f(x) = 1$$
$$\lim_{x\to 1^+} f(x) = 2$$
$$f(1) = 2$$
$$\lim_{x\to 2} f(x) = 1$$
$$f(2) = 0.5$$



Which one of the discontinuities is removable? Why?
Answer: The discontinuity at $x = 2$ is removable, since $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ both exist and are equal. The function can be made continuous at $x = 2$ by only redefining the value $f(2)$.
Give a definition of a *continuous* function. *See your textbook.*

(5) Let the function f be defined as follows.

$$f(x) = \begin{cases} 2x^2 + 5, & x < -1 \\ x + a, & x \geq -1. \end{cases}$$

Determine the value of a, which ensures that the function is continuous on its domain.
Answer: $a = 8$

(6) Use the Intermediate Value Theorem to show that the equation $x^2 + 5 = 2^x$ has a solution. Carefully explain how you applied the theorem. *Discussed in class.*

(7) Find the horizontal, vertical and oblique asymptotes of the following functions. Explain what they tell us about the behavior of the function when the independent variable is near a given point or approaches $\pm\infty$.
(a) $\dfrac{2x+6}{x^2-9} + 2$ (b) $\dfrac{x^2+3x-1}{x+1}$ *Answer:* (a) h.a. $y = 2$, v.a. $x = 3$, (b) v.a. $x = -1$, oblique asymptote $y = x+2$. The horizontal and oblique asymptotes give information about the behavior of the function as x approaches positive or negative infinity. If the function has a horizontal asymptote $y = c$, this means that the function approaches the value of c as the variable x approaches $\pm\infty$. On the other hand, if it has an oblique asymptote, the behavior of the function is close to the behavior of a line (with nonzero slope) as x approaches $\pm\infty$. The function has a vertical asymptote at $x = c$ if the function becomes unbounded as x approaches c from the left or from the right.

(8) Find the equation of the tangent line to the graph of $y = \frac{16}{x} - 2\sqrt[3]{x} + \sqrt{5}$ at $x = 8$.
Answer: $y = -\frac{5}{12}(x-8) - 2 + \sqrt{5}$

(9) Give examples of functions that are not differentiable. *Discussed in class.*

(10) Can a function be differentiable but not continuous? How about continuous, but not differentiable? *Discussed in class.*

(11) Find the derivative of the following functions.

(a) $f(t) = 3t^5 - \dfrac{\pi}{t^5} + \sqrt[3]{t^7}$ *Answer:* $f'(t) = 15t^4 + \frac{5\pi}{t^6} + \frac{7}{3}\sqrt[3]{t^4}$

(b) $g(s) = (2s + \dfrac{1}{s} + 3) \cdot (3s + 7s^2 - 1) \cdot (\sqrt[3]{\pi} - 21s^3)$ *Answer:* $g'(s) = (2 - \frac{1}{s^2}) \cdot (3s + 7s^2 - 1) \cdot (\sqrt[3]{\pi} - 21s^3) + (2s + \frac{1}{s} + 3) \cdot (3 + 14s) \cdot (\sqrt[3]{\pi} - 21s^3) + (2s + \frac{1}{s} + 3) \cdot (3s + 7s^2 - 1) \cdot (-63s^2)$

(c) $p(x) = \dfrac{(2x+1)(3x-1)}{4x^2 - x + 5}$ *Answer:* $p'(x) = \frac{(12x+1)(4x^2-x+5)-(6x^2+x-1)(8x-1)}{(4x^2-x+5)^2}$

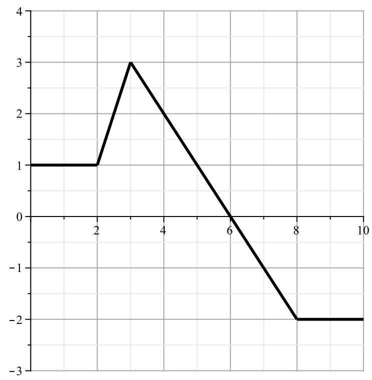
(12) At a time t the temperature of in a Petri dish as a function of time is given by $T(t) = t^3 - 6t^2 + 9t$, where T is measured in degrees Celsius, and t is measured in hours.

(a) Find the rate of change of temperature each time $T''(t) = 0$.
(b) What does it say about the temperature at time t if $T'(t) > 0$?
(c) What does it say about the temperature at time t if $T'(t) < 0$?
(d) What does it say about the rate of change of the temperature at time t if $T''(t) > 0$?

Discussed in class.

(13) An ant is moving along a straight line and its motion as a function of time as described in the graph below.

(a) Over what period of time is the ant moving the fastest?
(b) Over what period of time is the ant moving to the right/moving to the left/standing still?
(c) Graph the ant's velocity (where defined) as a function of time.



Discussed in class.