Review 2

MTH132-040, Calculus I

(1) Find the derivative of each of the following functions.

(a) $\sec(x^2+3)\sin(5x-1)$

(b) $\dfrac{\cos(\sin(x))}{x^5-3x+1}$

(c) $\tan(\cos((x^6-3x+8)^7))$

(2) Assume $f'(x) = h(x)\cdot g(x)$. Find $\dfrac{d}{dx}f(p(x))$ at $x=1$, given that

x	$g(x)$	$h(x)$	$p(x)$	$g'(x)$	$h'(x)$	$p'(x)$
1	5	-3	7	2	-9	3
3	2	6	-1	3	0	5
7	4	-5	2	3	-1	-2

(3) Find an equation of the tangent line to the curve $x^2y^2 = 9$ at the point $(-1, 3)$.

(4) When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

(5) Show that the linearization of $f(x) = (1+x)^k$ at $x = 0$ is $L(x) = 1 + kx$. Use this to find the linear approximation to $(1.02)^8$.

(6) State the *Extreme Value Theorem*. Give examples of functions which violate the theorem's assumptions and conclusion.

(7) For a continuous function f, defined on a closed interval $[a, b]$, where can the absolute extrema of the function occur?

(8) Find the absolute minimum and absolute maximum values of the following functions. Identify the intervals of increase and decrease.

(a) $f(x) = \sqrt{4-x^2}, \qquad -2 \le x \le 1$

(b) $g(t) = t^3 - 3t^2\ , \qquad x \in [-5, 0]$

(9) State *Rolle's Theorem*. Provide a graph to explain the theorem.

(10) State the *Mean Value Theorem*. Provide a graph to explain the theorem.

(11) Assume f is defined on the whole real line and let $f'(x) = \dfrac{(x-3)^2(x+1)}{\sqrt[3]{x+2}}$, $x \neq -2$. What are the critical points of f? On what intervals is f increasing or decreasing? At what points, if any, does f assume local minimum or local maximum values?

(12) Assume f is defined on the whole real line and let $f'(x) = (8x-5x^2)(4-x)^2$. What are the critical points of f? On what intervals is f increasing or decreasing? At what points, if any, does f assume local minimum or local maximum values? Where is the function concave up/ concave down? Sketch the graph of the function.

(13) Let $y = \dfrac{2x^2 + x - 1}{x^2 - 1}$.

(a) Find any vertical and horizontal asymptotes, as well as removable singularities, if any.
(b) Find the intervals of increase/decrease.
(c) Find the local extrema, if any.
(d) Find the intervals where the function is concave up/down.
(e) Find the inflection points, if any.
(f) Find the y-intercept.
(g) Sketch a possible graph of the function.

(14) Sketch the graph of a function f which satisfies the following conditions.

(a) f is twice differentiable for all x, except $x = 3$
(b) $f'(3)$ is not defined
(c) $f'(x) > 0$ on $(3, \infty)$
(d) $f'(x) < 0$ on $(-\infty, 3)$
(e) $f''(x) < 0$ for all x, except $x = 3$