Review 3

MTH132-040, Calculus I

(1) Jane is 2 miles offshore in a boat and wishes to reach a coastal village 6 miles down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?

Answer: $6 - \sqrt{\frac{16}{21}}$ miles away from the village.

(2) You are planning to make an open top rectangular box from an 8 inch by 15 inch piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way, and what is the volume?

Answer: $\frac{14}{3} \times \frac{35}{3} \times \frac{5}{3} in,\ V = \frac{2450}{27} in^3$

(3) Explain why Newton's method fails to find the root of the function $f(x) = \sqrt{x}$, no matter what initial guess we choose.

You can use a graph to show where x_2 is for a given initial guess, or recall that $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. Now note that $x_1 \geq 0$ and $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -x_1 < 0$, unless $x_1 = 0$. Thus x_2 is not in the domain of $f(x)$.

(4) Give a definition of an antiderivative of a function $f(x)$. Is it unique?

(5) Explain the difference between a *general* and a *particular* solution of a differential equation. Draw the general solution to $y' = 2x$ and the particular solution which satisfies $y(0) = -3$.

(6) Solve the following initial value problems.

(a) $\frac{dy}{dt} = \frac{1}{x^2} + x, \quad x > 0, \quad y(2) = 1$

Answer: $-\frac{1}{x} + \frac{x^2}{2} - \frac{1}{2}$

(b) $\frac{d^2r}{dt^2} = \frac{2}{t^3}, \quad \left.\frac{dr}{dt}\right|_{t=1} = 1, \quad r(1) = 1$

Answer: $r(t) = \frac{1}{t} + 2t - 2$

(7) You are driving along a highway at a steady 60 mph (88 ft/sec) when you see an accident ahead and slam on the breaks. What constant deceleration is required to stop your car in 242 ft?

Answer: $16 ft/sec^2$

(8) Assume the rate of change in number of bacteria on day t, $B'(t)$, is given by the following table.

Day	0	1	2	3	4
$B'(t)$	1	3	1	0	5

(a) Give a lower estimate of the total change in population of bacteria over the 4 days.

Answer: 2 bacteria

(b) Give an upper estimate of the total change in population of bacteria over the 4 days.

Answer: 12 bacteria

(c) Give an estimate of the total change in population of bacteria over the 4 days using 4 subintervals of length 1 with left-endpoint values.
Answer: 5 bacteria

(d) What is an upper estimate of the total population of bacteria on day 4, if there were 100 bacteria on day 1?
Answer: 112 bacteria

(9) Find the average temperature for Monday, if the temperature for that day as a function of time t was given by

$$T(t) = 40 + 10\sin(t), \qquad 0 \le t \le 24.$$

Answer: $\frac{5}{12}(97 - \cos(24))$

(10) Evaluate the following limit.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n^2}(5 + 3k)$$

Answer: $\frac{3}{2}$

(11) Express the following sum in sigma notation.

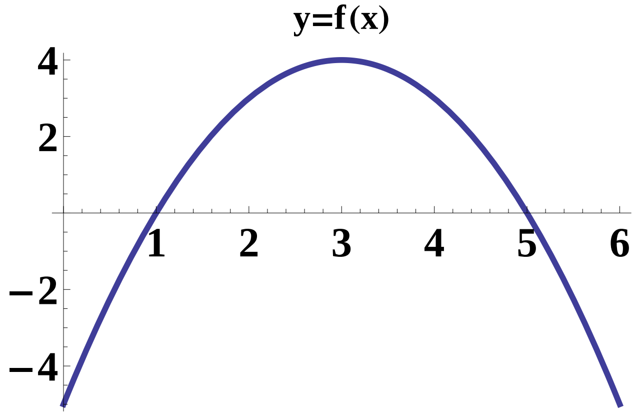
$$\frac{3}{2} - \frac{5}{4} + \frac{7}{8} - \frac{9}{16} + \frac{11}{32} - \frac{13}{64} + \frac{15}{128} - \frac{17}{256}$$

Answer: $\sum_{k=1}^{8} \frac{(-1)^{k+1}(2k+1)}{2^k}$

(12) Assume $\int_{-1}^{3} f(x)\,dx = 7$, $\int_{2}^{3} f(x)\,dx = 10$, $\int_{-1}^{2} g(x)\,dx = 4$. Find $\int_{-1}^{2} (3f(x) - 2g(x))\,dx$. Given this information, can one find $\int_{-1}^{2} f(x)^2\,dx$? How about $\left(\int_{-1}^{2} f(x)\,dx\right)^2$?

Answer: $\int_{-1}^{2} (3f(x) - 2g(x))\,dx = -17$. In general one cannot find $\int_{-1}^{2} f(x)^2\,dx$ using only the information given. On the other hand, $\left(\int_{-1}^{2} f(x)\,dx\right)^2 = 9$.

(13) Let $F(x)$ be defined on the interval $[0, 6]$ by $F(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(x)$ is given in the figure below. Use the figure to determine the intervals of increase and decrease



of $F(x)$. Can you determine if $F(1)$ is positive or negative? How about $F(4)$? Explain your reasoning.

(14) Find the derivative of each of the following functions.

(a) $f(s) = \displaystyle\int_{-5}^{3s} \left(5\sin^2(3x) + 7^x\right)\,dx$

<u>Answer:</u> $f'(s) = 3\left(5\sin^2(9s) + 7^{3s}\right)$

(b) $g(u) = \displaystyle\int_{-\sin(u)}^{\cos(u)} \left(e^{t^2} - 3t + 9\right)\,dt$

<u>Answer:</u> $g'(u) = -\left(e^{\cos^2(u)} - 3\cos(u) + 9\right)\sin(u) + \left(e^{\sin^2(u)} + 3\sin(u) + 9\right)\cos(u)$

(15) Evaluate the following indefinite integrals

(a) $\displaystyle\int \left(\sqrt{x} + 3x^2)(x+2)\right)\,dx$

<u>Answer:</u> $\frac{4}{3}x^{3/2} + \frac{2}{5}x^{5/2} + 2x^3 + \frac{3}{4}x^4 + C$

(b) $\displaystyle\int \left(\sin(5x) + \cos(x/3) + \pi - \sec^2(3x)\right)\,dx$

<u>Answer:</u> $-\frac{1}{5}\cos(5x) + 3\sin(x/3) + \pi x - \frac{1}{3}\tan(3x) + C$

(c) $\displaystyle\int \sin^5\frac{x}{3}\cos\frac{x}{3}\,dt$

<u>Answer:</u> $\frac{1}{2}\sin^6(x/3) + C$

(d) $\displaystyle\int \frac{1}{x^2}\sqrt{2 - \frac{1}{x}}\,dx$

<u>Answer:</u> $\frac{2}{3}\left(2 - \frac{1}{x}\right)^{3/2} + C$

(e) $\displaystyle\int s^3\sqrt{s^2+1}\,ds$

<u>Answer:</u> $\frac{1}{5}(1+x^2)^{5/2} - \frac{1}{3}(1+x^2)^{3/2} + C$

(16) Evaluate the following definite integrals

(a) $\displaystyle\int_{-1}^{1} (3x^2 - 4x + 7)\, dx$

Answer: 16

(b) $\displaystyle\int_{0}^{1} \frac{36}{(2x+1)^3}\, dx$

Answer: 8

(c) $\displaystyle\int_{0}^{\pi/2} \frac{3\sin x \cos x}{\sqrt{1 + 3\sin^2 x}}\, dx$

Answer: 1

(17) Find the area enclosed between $y = x^2$ and $y = -x^2 + 4x$.

Answer: 8/3

(18) Find the area enclosed between $x = 12y^2 - 12y^3$ and $x = y^2 - 2y$.

Answer: 4/3