# Supplement 1 for Section 2.2

**This material should come after Example 11 in Section 2.2 on page 53 of the text.**

**Example 12.** *For each* $x_0$,

**(a)** $\lim_{x\to x_0} \sin x = \sin x_0$ *and*

**(b)** $\lim_{x\to x_0} \cos x = \cos x_0$.

*Solution.*

$$\begin{aligned}
(\mathbf{a}) \lim_{x\to x_0} \sin x &= \lim_{x\to x_0} \sin(x - x_0 + x_0) \\
&= \lim_{x\to 0} \Big( \sin(x - x_0) \cos x_0 + \cos(x - x_0) \sin x_0 \Big) \\
&= \cos x_0 \lim_{x\to x_0} \sin(x - x_0) + \sin x_0 \lim_{x\to x_0} \cos(x - x_0).
\end{aligned}$$

As $x$ tends to $x_0$, the difference $x - x_0$ tends to 0. Consequently by the result of Example 11,

$$\lim_{x\to x_0} \sin(x - x_0) = 0 \text{ and } \lim_{x\to x_0} \cos(x - x_0) = 1.$$

Thus $\lim_{x\to x_0} \sin x = \sin x_0$. □

As similar argument using the formula for the cosine of the sum of two numbers can be used to prove (**b**).