Supplement 3 for Section 3.2

This material should after the proof of the Positive Integer Power Rule near the bottom of page 116.

Next we prove the Power Rule for positive integer roots. Recall that for a positive integer, n, the n^{th} root of x (for $x \geq 0$ if n is even and for any x is n is odd) is denoted by $\sqrt[n]{x}$ or $x^{\frac{1}{n}}$ and is the number whose n^{th} power is x; that is,

$$\left(\sqrt[n]{x}\right)^n = \left(x^{\frac{1}{n}}\right)^n = x \tag{1}$$

The same algebraic formula that was used in the previous proof, will be used but it a subtle fashion based on the equation (1).

Extended Power Rule. *Let n be a positive integer. Then* $\dfrac{d}{dx}x^{\frac{1}{n}} = \dfrac{1}{n}x^{\frac{1}{n}-1}$.

Proof. By definition

$$\begin{aligned}
\frac{d}{dx}x^{\frac{1}{n}} &= \lim_{z\to x}\frac{z^{\frac{1}{n}} - x^{\frac{1}{n}}}{z-x} = \lim_{z\to x}\frac{z^{\frac{1}{n}} - x^{\frac{1}{n}}}{\left(z^{\frac{1}{n}}\right)^n - \left(x^{\frac{1}{n}}\right)^n} \quad \text{(Note the use of (1).)} \\
&= \lim_{z\to x}\frac{z^{\frac{1}{n}} - x^{\frac{1}{n}}}{(z^{\frac{1}{n}} - x^{\frac{1}{n}})\left((z^{\frac{1}{n}})^{n-1} + (z^{\frac{1}{n}})^{n-2}x^{\frac{1}{n}}) + (z^{\frac{1}{n}})^{n-3}(x^{\frac{1}{n}})^2 + \cdots + (x^{\frac{1}{n}})^{n-1}\right)} \\
&= \frac{1}{nx^{\frac{n-1}{n}}} = \frac{1}{n}x^{\frac{1}{n}-1}
\end{aligned}$$

□

The Power Rule will be established for negative integer exponents in the next section and for rational exponents in Section 3.6.