# Supplement 4 for Section 3.3

**This material should come after the Quotient Rule on page 121.**

The Quotient Rule can be used to establish the Power Rule for negative integers; that is, for $n = -1, -2, -3, \ldots$.

**Further Extended Power Rule.** *Let $n$ be a negative integer. Then for any $x$*

$$\frac{d}{dx}x^n = nx^{n-1}$$

*Proof.* Let $n$ be a negative integer. Then $-n$ is a positive integer. Consequently by the Power Rule for Positive Integers, $\frac{d}{dx}x^{-n} = -nx^{-n-1}$. Thus

$$\begin{aligned}
\frac{d}{dx}x^n &= \frac{d}{dx}\frac{1}{x^{-n}} \text{(Recall that } \frac{1}{x^n} = x^{-n}.\text{)} \\
&= \frac{x^{-n}\frac{d}{dx}1 - 1\frac{d}{dx}x^{-n}}{(x^{-n})^2} \text{(By the Quotient Rule.)} \\
&= \frac{0 - (-nx^{-n-1})}{(x^{-2n})} = nx^{-n-n+2n} = nx^{n-1}
\end{aligned}$$

□