Give a careful proof of the following statement, using proof by contradiction.

PROPOSITION: For all $n \in \mathbb{Z}$, $n^2 + 2$ is not divisible by 4.

Proof Steps

1. Provide a precise negation of the original statement.

2. At the beginning of the proof, write the *hypothesis*: what is given or assumed, the setup of the proposition.

3. Leaving plenty of space on your proof page, at the bottom write the *conclusion*: what is to be deduced from the hypothesis setup.

4. Review the definition of each term in the hypothesis and conclusion. For example, what does "m is divisible by n" mean?

5. The body of the proof is a bridge of logical deductions from the hypothesis to the conclusion.

6. Finish off by explaining why the contradiction you arrived at implies the original proposition is true.

7. Once all this is done, do not just turn in the mess. Make a final clean draft with a clear logical flow, leaving out anything that was not needed.

8. Judge your audience: leave out details that will be obvious to them. As in other forms of expository writing, more words = more pain. We consider a proof rigorous not if it spells out every last detail, but rather if it is clear to the audience how every last detail could be supplied if wanted. Assume your audience is one of your classmates who has not done this assignment and needs a good deal of explanation. Do NOT assume the audience is the professor, who knows all about the problem and needs only a hint that you understand the proof.