Homework due October 9th, 2013

1. Let $G = \{f : \mathbb{R} \to \mathbb{R} \,|\, f \text{ is a bijective function}\}$. Show that the set G together with function composition satisfies the four group axioms. That is,

   A_1. **Closure:** $\forall f, g \in G,\ f \circ g \in G$.
   Is this statement true? Give a brief explanation - you can use previous knowledge. You do not need to provide a detailed proof.

   A_2. **Associativity:** $\forall f, g, h \in G,\ (f \circ g) \circ h = f \circ (g \circ h)$
   Is this statement true for functions? No proof is needed.

   A_3. **Identity element:** $\exists e \in G$ such that $\forall f \in G,\ f \circ e = e \circ f = f$.
   What function corresponds to the identity element? Does it belong to the set G?

   A_4. **Inverse element:** $\forall f \in G, \exists g \in G$ such that $f \circ g = g \circ f = e$.
   What function corresponds to the inverse element of f? Does it belong to the set G? Why?

   Is it true that G together with function composition satisfies the following additional property:
   Commutativity: $\forall f, g \in G, f \circ g = g \circ f$?
   Support your answer by an example.

2. Let the following abstract system be given.

   Undefined terms: Gub, rok

   A_1. There exist exactly five gubs.

   A_2. Each rok is a subset of those five gubs.

   A_3. There exist exactly two roks.

   A_4. Each rok contains at least two gubs.

Which of the following models satisfy the above system? If a model does not satisfy the axiomatic system, explain which of the axioms is violated.

Model 1. The set of gubs is given by $\{A, B, C, D, E\}$. The set of roks is given by $\{\{A, B\}, \{B, C, D\}\}$

Model 2. The set of gubs is given by $\{1, 2, 3, 4, 5\}$. The set of roks is given by $\{\{1, 2\}, \{2, 3, 4, 5\}\}$

Model 3. The set of gubs is given by $\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit, \triangle\}$. The set of roks is given by $\{\{\clubsuit\}, \{\diamondsuit, \heartsuit, \spadesuit\}\}$

Model 4 is given by the figure below. Which of the above models is it equivalent to? Write the one-to-one correspondence between the elements of the two models.