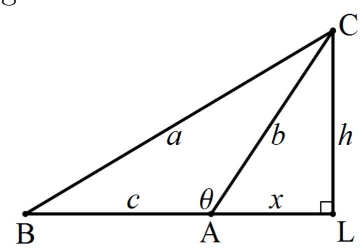
Math 299 **Supplement: Houston Ch. 3** **Aug 28, 2013**

We improve the proof of the Law of Cosines for an obtuse triangle on p. 23, following the suggestions in Ch. 3.

THEOREM: Let triangle $\triangle ABC$ have opposite side-lengths a, b, c, and an obtuse angle $\theta > 90^\circ$ at A. Then:

$$a^2 = b^2 + c^2 - 2bc\cos\theta.$$

Proof: Let $\overline{CL}$ be the altitude perpendicular to line $\overleftrightarrow{AB}$, let h be the length CL, and let x be the length AL:



We apply Pythagoras' Theorem twice, first to the right triangle $\triangle ACL$:

$$b^2 = x^2 + h^2.$$

Applying it to the right triangle $\triangle BCL$, we obtain:

$$\begin{aligned} a^2 &= (c+x)^2 + h^2 \\ &= c^2 + 2cx + x^2 + h^2 \\ &= c^2 + 2cx + b^2, \end{aligned}$$

after substituting the first formula.

By definition, the cosine of the acute external angle $\angle CAL$ is $\cos(180-\theta) = x/b$, so:

$$x = b\cos(180-\theta) = -b\cos\theta.$$

Substituting this for x in the earlier equation, we deduce the desired formula:

$$a^2 = c^2 + 2c(-b\cos\theta) + b^2 = b^2 + c^2 - 2bc\cos\theta.$$