

Problem 1. Let $R_0 = \{x \in \mathbb{Z} \mid x \equiv 0 \pmod{3}\}$, $R_1 = \{x \in \mathbb{Z} \mid x \equiv 1 \pmod{3}\}$ and $R_2 = \{x \in \mathbb{Z} \mid x \equiv 2 \pmod{3}\}$. Determine if the following statements are true or false. Justify your answer with a proof.

1. $\forall n \in R_0, n^2 \in R_1$.
2. $\forall n \in R_1, n^2 \in R_1$.
3. $\forall n \in R_2, n^2 \in R_2$.
4. $\forall n \in \mathbb{Z}, n^2 \notin R_2$.
5. $\forall n \in R_1 \cup R_2 \exists m \in \mathbb{Z}, mn \in R_1$.