
2.40 In each of the following, two open sentences $P(x, y)$ and $Q(x, y)$ are given, where the domain of both x and y is $\mathbb{Z}$. Determine the truth value of $P(x, y) \iff Q(x, y)$ for the given values of x and y.

(a) $P(x, y) : x^2 - y^2 = 0$ and $Q(x, y) : x = y$. $(x, y) \in \{(1, -1), (3, 4), (5, 5)\}$.

(b) $P(x, y) : |x| = |y|$ and $Q(x, y) : x = y$. $(x, y) \in \{(1, 2), (2, -2), (6, 6)\}$.

(c) $P(x, y) : x^2 + y^2 = 1$ and $Q(x, y) : x + y = 1$. $(x, y) \in \{(1, -1), (-3, 4), (0, -1), (1, 0)\}$.

2.44 (Bonus) Let $S = \{1, 2, 3, 4\}$. Consider the following open sentences over the domain S:

$$P(n) : \frac{n(n-1)}{2} \text{ is even.}$$

$$Q(n) : 2^{n-2} - (-2)^{n-2} \text{ is even.}$$

$$R(n) : 5^{n+1} + 2^n \text{ is prime.}$$

Determine four distinct elements a, b, c, d in S such that all of the following are satisfied.

(i) $P(a) \Rightarrow Q(a)$ is false;

(ii) $Q(b) \Rightarrow P(b)$ is true;

(iii) $P(c) \iff R(c)$ is true;

(iv) $Q(d) \iff R(d)$ is false.

2.46 For statements P and Q, show that $P \Rightarrow (P \vee Q)$ is a tautology.

2.52 Let P and Q be statements.

(a) Is $\sim (P \vee Q)$ logically equivalent to $(\sim P) \vee (\sim Q)$? Explain.

(b) What can you say about the biconditional $\sim (P \vee Q) \iff ((\sim P) \vee (\sim Q))$?

2.54 For statements P and Q, show that $(\sim Q) \implies (P \wedge (\sim P))$ and Q are logically equivalent.

2.58 Verify the following laws stated in Theorem 2.18:

(a) Let P, Q, and R be statements. Then

$$P \vee (Q \wedge R) \text{ and } (P \vee Q) \wedge (P \vee R) \text{ are logically equivalent.}$$

(b) Let P and Q be statements. Then

$$\sim (P \vee Q) \text{ and } (\sim P) \wedge (\sim Q) \text{ are logically equivalent.}$$

2.60 Consider the implication: If x and y are even, then xy is even.

(b) State the converse of the implication.

(c) State the implication as a disjunction (see Theorem 2.17).

(d) State the negation of the implication as a conjunction (see Theorem 2.21(a)).