Math 299 **Homework Assignment, Due 03/12**

Prove each statement using mathematical induction. You might need to use strong induction for some of them.

1. $\sum_{x=1}^{n} \frac{1}{\sqrt{x}} \leq 2\sqrt{n}$.

2. $(2^{2n-1} + 1)$ is divisible by 3 $\forall n \in \mathbb{N}$.

3. The sum of cubes of three consecutive natural numbers is divisible by 9.

4. **(6.42)** A sequence $\{a_n\}$ is defined recursively by $a_1 = 1$, $a_2 = 2$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Conjecture an explicit formula for a_n and verify that your conjecture is correct.

5. **(6.44)** Consider the sequence F_1, F_2, F_3, ..., where

$$F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_5 = 8, ...$$

The terms of this sequence are called **Fibonacci numbers**.

   (a) Define the sequenc of Fibonacci numbers by means of a recurrence relation.

   (b) Prove that $2 \mid F_n$ if and only if $3 \mid n$.