
Section 1.1

1.2 Let $S = \{-2, -1, 0, 1, 2, 3\}$. Describe each of the following sets as $\{x \in S : p(x)\}$, where $p(x)$ is some condition on x.

(a) $A = \{1, 2, 3\}$

(b) $B = \{0, 1, 2, 3\}$

(c) $C = \{-2, -1\}$

(d) $D = \{-2, 2, 3\}$.

1.4 Write each of the following sets by listing its elements within braces.

(a) $A = \{n \in \mathbb{Z} : -4 < n \leq 4\}$

(b) $B = \{n \in \mathbb{Z} : n^2 < 5\}$

(c) $C = \{n \in \mathbb{N} : n^3 < 100\}$

(d) $D = \{x \in \mathbb{R} : x^2 - x = 0\}$

(e) $E = \{x \in \mathbb{R} : x^2 + 1 = 0\}$

1.8 Let $A = \{n \in \mathbb{Z} : 2 \leq |n| < 4\}$, $B = \{x \in \mathbb{Q} : 2 < x \leq 4\}$,
$C = \{x \in \mathbb{R} : x^2 - (2 + \sqrt{2})x + 2\sqrt{2} = 0\}$, and $D = \{x \in \mathbb{Q} : x^2 - (2 + \sqrt{2})x + 2\sqrt{2} = 0\}$.

(a) Describe the set A by listing its elements.

(b) Give an example of three elements that belong to B but do not belong to A.

(c) Describe the set C by listing its elements.

(d) Describe the set D in another manner.

(e) Determine the cardinality of each of the sets A, C and D.

1.68 Let $S = \{-10, -9, ..., 9, 10\}$. Describe each of the following sets as $\{x \in S : p(x)\}$, where $p(x)$ is some condition on x.

(a) $A = \{-10, -9, ..., -1, 1, ..., 9, 10\}$

(b) $B = \{-10, -9, ..., -1, 0\}$

(c) $C = \{-5, -4, ..., 0, 1, ..., 7\}$

(d) $D = \{-10, -9, ..., 4, 6, 7, ..., 10\}$.

Section 1.2

1.12 Which of the following sets are equal?

$$A = \{n \in \mathbb{Z} : |n| < 2\}, \qquad B = \{n \in \mathbb{Z} : n^3 = n\}, \qquad C = \{n \in \mathbb{Z} : n^2 \leq n\},$$
$$D = \{n \in \mathbb{Z} : n^2 \leq 1\}, \qquad E = \{-1, 0, 1\}.$$

1.14 Find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$ for

(a) $A = \{1, 2\}$

(b) $B = \{\emptyset, 1, \{a\}\}$.

Section 1.3

1.22 Let $U = \{1, 3, ..., 15\}$ be the universal set, $A = \{1, 5, 9, 13\}$ and $B = \{3, 9, 13\}$. Determine the following:

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

(e) $\bar{A}$

(f) $A \cap \bar{B}$.

1.26 Let U be a universal set and let A and B be two subsets of U. Draw a Venn diagram for each of the following sets:

(a) $\overline{A \cup B}$

(b) $\bar{A} \cap \bar{B}$

(c) $\overline{A \cap B}$

(d) $\bar{A} \cup \bar{B}$.

What can you say about parts (a) and (b)? parts (c) and (d)?

1.30 Let $A = \{x \in \mathbb{R} : |x - 1| \leq 2\}$, $B = \{x \in \mathbb{R} : |x - 1| \geq 1\}$ and $C = \{x \in \mathbb{R} : |x + 2| \leq 3\}$.

(a) Express A, B and C using interval notation.

(b) Determine each of the following sets using interval notation:

$$A \cup B, \qquad A \cap B, \qquad B \cap C, \qquad B - C.$$

1.76 Which of the following sets are equal?

$A = \{n \in \mathbb{Z} : -4 \leq n \leq 4\}$, $B = \{x \in \mathbb{N} : 2x + 2 = 0\}$, $C = \{x \in \mathbb{Z} : 3x - 2 = 0\}$,
$D = \{x \in \mathbb{Z} : x^3 = 4x\}$, $E = \{-2, 0, 2\}$.

Section 1.4

1.36 For a real number r, define S_r to be the interval $[r - 1, r + 2]$. Let $A = \{1, 3, 4\}$. Determine $\bigcup_{\alpha \in A} S_\alpha$ and $\bigcap_{\alpha \in A} S_\alpha$.

1.38 For a real number r, define $A_r = \{r^2\}$, B_r as the closed interval $[r - 1, r + 1]$, and C_r as the interval (r, ∞). For $S = \{1, 2, 4\}$, determine

(a) $\bigcup_{\alpha \in S} A_\alpha$ and $\bigcap_{\alpha \in S} A_\alpha$

(b) $\bigcup_{\alpha \in S} B_\alpha$ and $\bigcap_{\alpha \in S} B_\alpha$

(c) $\bigcup_{\alpha \in S} C_\alpha$ and $\bigcap_{\alpha \in S} C_\alpha$.

Section 1.5

1.46 Which of the following are partitions of $A = \{a, b, c, d, e, f, g\}$? For each collection of sets that is not a partition of A, explain your answer:

(a) $S_1 = \{\{a, c, e, g\}, \{b, f\}, \{d\}\}$

(b) $S_2 = \{\{a, b, c, d\}, \{e, f\}\}$

(c) $S_3 = \{A\}$

(d) $S_4 = \{\{a\}, \emptyset, \{b, c, d\}, \{e, f, g\}\}$

(e) $S_5 = \{\{a, c, d\}, \{b, g\}, \{e\}, \{\ b, f\}\}$.

1.50 Give an example of a partition of $\mathbb{N}$ into three subsets.

Section 1.6

1.59 For $A = \{a, b\}$, determine $A \times \mathcal{P}(A)$.

1.64 For $A = \{1, 2\}$ and $B = \{1\}$, determine $\mathcal{P}(A \times B)$.

1.66 - bonus For $A = \{a \in \mathbb{R} : |a| \leq 1\}$ and $B = \{b \in \mathbb{R} : |b| = 1\}$, give a geometric description of the points in the xy-plane belonging to $(A \times B) \cup (B \times A)$.

1.72 Let $U = \{1, 2, 3\}$ be the universal set and let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{1, 3\}$. Determine the following:

(a) $(A \cup B) - (B \cap C)$

(b) $\overline{A}$

(c) $\overline{B \cup C}$

(d) $A \times B$.