
READ THIS FIRST: *This homework assignment is different. After your solutions are graded and returned, you will be asked to make corrections, revise your solutions, and re-submit the entire assignment. Your revised solutions will again be graded. So, effectively, this assignment is worth double points.*

The purpose of the "submit, revise, re-submit" process is to provide you with an opportunity to work on improving your mathematical writing. Your writing will be carefully scrutinized and the grade you earn will reflect this.

Directions: Read Chapter 7. Then write a solution to each of the exercises below. Each solution must begin with a complete and accurate restatement of the exercise. These exercises are modified versions of some of the exercises in Chapter 7.

1. Two recursively defined sequences $\{a_n\}$ and $\{b_n\}$ of positive integers have the same recurrence relation: for each $n \geq 3$,

   $$a_n = 2a_{n-1} + a_{n-2} \quad \text{and} \quad b_n = 2b_{n-1} + b_{n-2}.$$

   The initial values for $\{a_n\}$ are $a_1 = 1$ and $a_2 = 3$, whereas the initial value for $\{b_n\}$ are $b_1 = 1$ and $b_2 = 2$.

   Determine whether each of the following conjectures is true or false.

   Conjecture A: $a_n = 2^{n-2} \cdot n + 1$ for every integer $n \geq 2$.

   Conjecture B: $b_n = \frac{1}{2\sqrt{2}}[(1+\sqrt{2})^n - (1-\sqrt{2})^n]$ for every integer $n \geq 2$.

2. Express the statement below in symbols (for example, using the symbols $\exists$, $\forall$, $\implies$, $\vee$, $\wedge$, $\iff$, and $\sim$). Then prove the statement.

   For every positive real number a and positive rational number b, there exist a real number c and irrational number d such that $ac + bd = 1$.

3. Prove or disprove: for every two sets A and B, we have that $(A \cup B) - B = A$.

4. Prove or disprove: for every rational number a/b such that $a, b \in \mathbb{N}$, there exists a rational number c/d such that c and d are positive odd integers and $0 < c/d < a/b$.

5. Prove or disprove: there exist positive integers x and y such that $x^2 - y^2 = 101$.