
Section 8.1

8.4 Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then $R_1 = \{(a, 2), (a, 3), (b, 1), (b, 3), (c, 4)\}$ is a relation from A to B while $R_2 = \{(1, b), (1, c), (2, a), (2, b), (3, c), (4, a), (4, c)\}$ is a relation from B to A. A relation R is defined on A by $x\,R\,y$ if there exists $z \in B$ such that $x\,R_1\,z$ and $z\,R_2\,y$. Express R by listing its elements.

8.6 A relation R is defined on $\mathbb{N}$ by $a\,R\,b$ if $a/b \in \mathbb{N}$. For $c, d \in \mathbb{N}$, under what conditions is $c\,R^{-1}\,d$?

8.10 Let A be a set with $|A| = 4$. What is the maximum number of elements that a relation R on A can contain so that $R \bigcap R^{-1} = \emptyset$?

Section 8.2

8.12 Let $S = \{a, b, c\}$. Then $R = \{(a, a), (a, b), (a, c)\}$ is a relation on S. Which of the properties reflexive, symmetric and transitive does the relation R possess? Justify your answer.

8.14 Let $A = \{a, b, c, d\}$. Give an example (with justification) of a relation R on A that has **none** of the following properties: reflexive, symmetric, transitive.

8.16 Let $A = \{a, b, c, d\}$. How many relations defined on A are reflexive, symmetric and transitive and contain the ordered pairs (a, b), (b, c), (c, d)?

8.22 Let S be the set of all polynomials of degree at most 3. An element $s(x)$ of S can be expressed as $s(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$. A relation R is defined on S by $p(x)\,R\,q(x)$ if $p(x)$ and $q(x)$ have a real root in common. (For example $p(x) = (x - 1)^2$ and $q(x) = x^2 - 1$ have the root 1 in common so that $p(x)\,R\,q(x)$.) Determine which of the properties reflexive, symmetric and transitive are possessed by R.

Section 8.3

8.24 Let R be an equivalence relation on $A = \{a, b, c, d, e, f, g\}$ such that $a\,R\,c$, $c\,R\,d$, $d\,R\,g$ and $b\,R\,f$. If there are three distinct equivalence classes resulting from R, then determine these equivalence classes and determine all elements of R.