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**Section 8.3**

**8.28** (a) Let $R$ be a relation defined on $\mathbb{Z}$ by $a\,R\,b$ if $a+b$ is even. Show that $R$ is an equivalence relation and determine the distinct equivalence classes.

(b) Suppose that "even" is replaced by "odd" in (a). Which of the properties reflexive, symmetric and transitive does $R$ possess?

**8.30** Let $H = \{2^m : m \in \mathbb{Z}\}$. A relation $R$ is defined on the set $\mathbb{Q}^+$ of positive rational numbers by $a\,R\,b$ if $a/b \in H$.

(a) Show that $R$ is an equivalence relation.

(b) Describe the elements in the equivalence class $[3]$.

**Section 8.4**

**8.38** Let $R$ be a relation defined on the set $\mathbb{N}$ by $a\,R\,b$ if $a \mid 2b$ or $b \mid 2a$. Prove or disprove: $R$ is an equivalence relation.

**Section 8.5**

**8.52** Let $R$ be defined on $\mathbb{Z}$ by $a\,R\,b$ if $a^2 \equiv b^2 \pmod 5$. Prove that $R$ is an equivalence relation and determine the distinct equivalence classes.

**Section 8.6**

**8.54** Construct the addition and multiplication tables for $\mathbb{Z}_5$.

**8.56** In $\mathbb{Z}_{11}$, express the following sums and products as $[r]$, where $0 \le r < 11$.

(a) $[7]+[5]$

(b) $[7]\cdot[5]$

(c) $[-82]+[207]$

(d) $[-82]\cdot[207]$

**8.58** Prove that the multiplication in $\mathbb{Z}_n$, $n \ge 2$, defined by $[a][b]=[ab]$ is well defined. (See result 4.11.)