
Section 9.3

9.20 A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined by $f(n) = 2n + 1$. Determine whether f is (a) injective, (b) surjective.

9.63 Let function $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 + 3x + 4$.

(a) Show that f is not injective.

(b) Find all pairs of real numbers such that $f(r_1) = f(r_2)$.

(c) Show that f is not surjective.

(d) Find the set S of all real numbers such that if $s \in S$, then there is no real number x such that $f(x) = s$.

(e) What well-known set is the set S in (d) related to?

9.67 For each of the following functions, determine, with explanation, whether the function is one-to-one and whether it is onto.

(c) $h : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$, where $h(r, s) = (2r + 1, 4s + 3)$

(d) $\phi : \mathbb{Z} \times \mathbb{Z} \to S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$, where $\phi(a, b) = a + b\sqrt{2}$

9.68 (Bonus) Let S be a nonempty set. Show that there exists an injective function from $\mathcal{P}(S)$ to $\mathcal{P}(\mathcal{P}(S))$.

9.78 (Bonus) A function $F : \mathbb{N} \to \mathbb{N} \cup \{0\}$ is defined by $F(n) = m$ for each $n \in \mathbb{N}$, where m is that nonnegative integer for which $3n + 1 = 2^m k$ and k is an odd integer. Prove or disprove the following:

(a) F is one-to-one.

(b) F is onto.