
Section 9.4

9.31 Let $f : \mathbb{Z}_5 \to \mathbb{Z}_5$ be a function defined by $f([a]) = [2a + 3]$.

(a) Show that f is well-defined.

(b) Determine whether f is bijective.

9.32 Prove that the function $f : \mathbb{R} - \{2\} \to \mathbb{R} - \{5\}$ defined by $f(x) = \dfrac{5x+1}{x-2}$ is bijective.

9.34 Give a proof of Theorem 9.7 using mathematical induction.

Theorem 9.7 If A and B are finite sets with $|A| = |B| = n$, then there are $n!$ bijective functions from A to B.

9.36 (Bonus) Let $A = \{a, b, c, d, e, f\}$ and $B = \{u, v, w, x, y, z\}$. With each element $r \in A$, there is associated a lost or subset $L(r) \subseteq B$. The goal is to define a "list function" $\phi : A \to B$ with the property that $\phi(r) \in L(r)$ for each $r \in A$.

(a) $L(a) = \{w, x, y\}$, $L(b) = \{u, z\}$, $L(c) = \{u, v\}$, $L(d) = \{u, w\}$, $L(e) = \{u, x, y\}$, $L(f) = \{v, y\}$, does there exist a bijective list function $\phi : A \to B$ for these lists?

(b) $L(a) = \{u, v, x, y\}$, $L(b) = \{v, w, y\}$, $L(c) = \{v, y\}$, $L(d) = \{u, w, x, z\}$, $L(e) = \{v, w\}$, $L(f) = \{w, y\}$, does there exist a bijective list function $\phi : A \to B$ for these lists?