
Section 9.6

9.54 The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x+3$ and $g(x) = -3x+5$.

(a) Show that f is one-to-one and onto.

(b) Show that g is one-to-one and onto.

(c) Determine the composition function $g \circ f$.

(d) Determine the inverse functions f^{-1} and g^{-1}.

(e) Determine the inverse function of $(g \circ f)^{-1}$ of $g \circ f$ and the composition $f^{-1} \circ g^{-1}$.

9.57 The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if} \quad x < 1 \\ \sqrt{x-1} & \text{if} \quad x \geq 1 \end{cases}$$

(a) Show that f is a bijection.

(b) Determine the inverse function f^{-1} of f.

9.81 The function $h : \mathbb{Z}_{16} \to \mathbb{Z}_{24}$ is defined by $h([a]) = [3a]$ for $a \in \mathbb{Z}$.

(a) Prove that the function h is well defined; that is, prove that if $[a] = [b]$ in $\mathbb{Z}_{16}$, then $h([a]) = h([b])$ in $\mathbb{Z}_{24}$.

(b) For the subsets $A = \{[0], [3], [6], [9], [12], [15]\}$ and $B = \{[0], [8]\}$ of $\mathbb{Z}_{16}$, determine the subsets $h(A)$ and $h(B)$ of $\mathbb{Z}_{24}$.

(c) For the subsets $C = \{[0], [6], [16], [18]\}$ and $D = \{[4], [8], [16]\}$ of $\mathbb{Z}_{24}$, determine the subsets $h^{-1}(C)$ and $h^{-1}(D)$.