
In general, to prove two sets A and B are equal, we need to show that both $A \subseteq B$ and $B \subseteq A$ are true.

$A \subseteq B : \forall x \in A, x \in B.$

EXAMPLES.

(1H) $A - B = A \cap \overline{B}$

(2) $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

(3H) Let $A = \{x : x \equiv 1\,(mod\,4)\}$ and $B = \{x : x \equiv 1\,(mod\,2)\}$.
Prove that $A \subset B$.

(4) Let $A = \{x : x \equiv 0\,(mod\,2)\}$ and $B = \{x : x \equiv 0\,(mod\,3)\}$ and $C = \{x : x \equiv 0\,(mod\,6)\}$. Prove that $C \subseteq A \cap B$.
(We will later prove $C = A \cap B$.)

Exercises Let A, B and C be subsets of the universal set U. Prove the following.

(1) If $A \subseteq B$, then $A \cap B = A$.

(2H) $A \cup B = A$ if and only if $B \subseteq A$.

(3H) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

(4) $A - (B \cup C) = (A - B) \cap (A - C)$.

Fundamental Properties of Set Operations For sets A, B and C,

1. *Commutative Laws*

   (1) $A \cup B = B \cup A$ *(textbook)*

   (2) $A \cap B = B \cap A$

2. *Associative Laws*

   (1) $A \cup (B \cup C) = (A \cup B) \cup C$

   (2) $A \cap (B \cap C) = (A \cap B) \cap C$

3. *Distributive Laws*

   (1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ *(textbook)*

   (2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. *De Morgan's Laws*

   (1) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ *(textbook)*

   (2) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ *(example)*

Proofs Involving Cartesian Products of Sets Let A, B, C and D be subsets of the universal set U. Prove the following.

1. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

   (Remark: $(A \times C) \cup (B \times D) \neq (A \cup B) \times (C \cup D)$.)

2. If $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$. (*textbook*)

3. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

4. $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

5. $A \times (B - C) = (A \times B) - (A \times C)$. (*textbook*)

6. (Challenge) $\overline{(A \times B)} = (\overline{A} \times \overline{B}) \cup (\overline{A} \times B) \cup (A \times \overline{B})$.

Exercises

1. $(A \cap B) \cap (A^c \cup B^c) = \emptyset$. (*This is a proof by contradiction, so we will revisit this problem later*)

2. $(A \cap B) \cup (A^c \cup B^c) = U$. (*Proof this statement directly by using the definitions, not the fundamental properties*)